RESPONSE OF CABLE-STAYED MASTS TO TURBULENT WIND

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INTRODUCTION:

Cable-staying has become popular in controlling the otherwise large displacements that can occur in tall slender vertical masts from buffeting action in high winds. Whilst the benefits to the performance of telecommunications devices of a stiffer structural system are immediately obvious for such masts the introduction of cables poses some problems in the modelling of their displacement response because of non-linear features that also result from the use of cables.

These features would include:

(i) Non-linear stiffness of the guy clusters which are very sensitive to the level of initial guy tension and are also direction-dependent (see Fig. 1).

(ii) The effect of axial load due to the guys in modifying the stiffness of the mast section.

(iii) Wind loading effects on the guys, dynamic interaction effects and possible aerodynamic instability effects.

(iv) Non-negligible acrosswind response which, when compared with the alongwind direction of response, is characterised by: a zero mean wind component, only half of the associated aerodynamic damping level (for comparable dynamic conditions) and significantly lower guy cluster stiffnesses which together normally result in smaller levels of overall response than for the alongwind direction.

Whilst item (i) is sometimes considered in the modelling of cable-stayed masts, items (iii) to (iv) are seldom incorporated in such modelling.

In addition, the introduction of cables often complicates the character of the resultant dynamic response of an otherwise free-standing mast as higher order modes than the primary may now contribute significantly to this response and can no longer be neglected in the final design solution.

However, at least at the preliminary design stage, some immediate computational benefits can arise in adopting a linear solution as spectral-based gust factor methods can be used to provide an estimate of peak response parameters even when several modes may be contributing to this response. (Recent developments in this area may be found in [1], [2] and [3]).

Application of these methods requires knowledge of the participating modes, (both frequency and mode shape) as well as damping levels for which contributions due to aerodynamic damping may dominate. This is normally achieved via an eigenvalue analysis on a mathematical model of the mast system in which the tangent stiffness matrix at the mean wind displaced position of the mast is used in combination with a mass matrix that contains contributions from the guy clusters as well as antennae and other attachments.

Closed form derivations may however be possible when only the first mode is considered in such analyses allowing for expedient application of these techniques, at the expense of accuracy.

When it comes to the final design however, (especially in the case of the taller, more slender mast solutions that may incorporate several levels of guy-staying), a non-linear time history response analysis may be necessary in order to more reasonably account for the influence of the guy cluster system adopted and its affect on the resultant response of the mast to turbulent wind. Whilst some attention has been given to such modelling by various researchers in recent times ([4], [5], [6], [7]) there does not appear to be any particular method that has been favoured in the literature, although non-linear finite-element based approaches seem to be the most popular. (Differences are to be found not only in the manner in which the cable elements are modelled and the degree to which non-linear features are accounted for, but also in the methods used to describe the dynamic loading from wind).

This paper therefore attempts to address two widely different elements in the modelling of the response of tall guy-stayed masts to the effects of buffeting by wind, viz

(1) A simple technique for estimating the first mode natural frequency of such masts (for use with gust factor methods).

(2) A technique for modelling wind turbulence at selected locations along the mast section suitable for incorporation into a non-linear time history response analysis procedure.

FIRST MODE NATURAL FREQUENCY OF A GUYED-MAST.

Let the response x(t) of a mast be characterised by its first mode shape, $\Phi(y)$, at position y above the ground and a circular frequency, ω.

Hence
$$x(t) = \Phi(y) \sin \omega t \tag{1}$$
and
$$\dot{x}(t) = \Phi(y) \cos \omega t \tag{2}$$

Now if there are N_m discrete point masses on the mast section which itself is assumed to have a mass per unit length $m(y)$, flexural rigidity $EI(y)$ and overall height, h, then the maximum kinetic energy KE_{max} occurs when $\cos \omega t = 1$, viz,

$$KE_{max} = \frac{1}{2}\omega^2 \left(\int_0^h m(y)\, \Phi^2(y)\, dy + \sum_{j=1}^{N_m} M_j\, \Phi^2(y_j) \right) \tag{3}$$

where discrete masses M_j are located at positions y_j along the mast.

The maximum potential energy, PE_{max}, is composed of a flexural contribution due to the mast section and contributions due the cable clusters, modelled as linear springs with a tangent stiffness corresponding to the displaced position of the mast at the design mean wind speed; viz

$$PE_{max} = \frac{1}{2} \left(\int_0^h EI(y) \frac{d^2\Phi(y)}{dy^2}\, dy + \sum_{i=1}^{N_g} k_i\, \Phi^2(y_i) \right) \tag{4}$$

where N_g is the number of guy cluster levels and y_i the positions of these levels.

Rayleigh's method, which is based upon the principle of conservation of energy, equates KE_{max} to PE_{max}, which for a nominated mode shape then allows a solution to the first mode natural circular frequency, ω.

For a mast that is considered "tuned" (i.e. guy cluster stiffnesses have been chosen to produce a pre-defined lean in the mast section under mean wind conditions) and for which the mast section is considered slender ($EI(y_i)/k_i\, y_i^3$ is small), then, for a parabolic mode shape:

$$\Phi(y) = \alpha\, y^2 \tag{5}$$

equating (3) to (4) results in the approximation for ω given by

$$\omega \approx 1.12 \sqrt{\frac{(k_1 + k_{N_g}) \left(\frac{N_g + 1}{N_g}\right)}{M_T + 5 \sum_{j=1}^{N_m} M_j \left(\frac{y_j}{h}\right)^4}} \tag{6}$$

where M_T is the total mass of the mast section (which includes contributions due to the cable clusters corresponding to half their total weight) and k_i and k_{N_g} are the guy stiffnesses in the alongwind direction at the first and last guy level positions respectively.

SIMULATING ALONGWIND TURBULENCE

Several techniques for simulating alongwind turbulence traces both for their spectral content and their correlation characteristics have been described in the literature [8],[9]. In the case of cable-stayed masts such traces can be generated at selected positions along the mast section using these techniques then transformed to wind force.

A summary of a technique that has been found to be particularly versatile and expedient by the author is provided below:

1. Select an appropriate spectral density variation, $S_v(f)$, for the wind speed fluctuations pertaining to the site under consideration (eg. Davenport, Harris, Kolmogorov etc.)
2. Allow for a contribution by mean wind speed that varies with height according to a power law profile.
3. Consider the coherence relationship between wind fluctuation traces at positions i and j to be described by

$$\rho_{ij}^{2}(f) = \exp^{-\left(\frac{C\,f\,d_{ij}}{\bar{V}_z}\right)} \qquad (7)$$

   where d_{ij} is the vertical distance of separation between the traces, $\bar{V}_z$ is the mean wind speed at a height z centrally located between positions z_i and z_j, f is frequency and C is a constant (usually taken to be around 10).

4. Select K sequential points along the mast section where wind speed traces are required to be generated, a fixed time step value of dt and a total time length of record T where T = Ndt and where N is a suitable number (highest prime factor ≤23 for the FFT software available to this author).
5. Produce a set of N/2 random phase angles (0-2π) for the first trace, viz $\phi_1(f_n)$ that would apply to frequency values at increments of 1/T to a Nyquist frequency of 1/2dt, (i.e. $f_n = n/T$).
6. Select a phase shift between levels i - 1 and current level i that satisfies:

$$\cos(\Delta\phi_{i-1,i}(f_n)) = \rho_{i-1,i}(f_n) \qquad (8)$$

   allowing for both positive and negative values of this phase shift to be equally likely to produce $\phi_i(f_n)$ by simply adding onto $\phi_{i-1}(f_n)$.

7. Use an Inverse Fast Fourier Transform on Fourier coefficients (a_n, b_n), produced at equal frequency increments $(df = 1/T)$ for $f_n = n/T$ via

$$(a_n, b_n) = \sqrt{2\,S_v(f_n)\,df}\;\chi_a(f_n)\;(\cos(\phi_i(f_n)), \sin(\phi_i(f_n)) \qquad (9)$$

   where $\chi_a^2(f_n)$ is the 'aerodynamic admittance' function applying to exposed area A_i to generate traces of fluctuations $v_i(t_n)$ where $t_n = n\,dt$ over N points.

8. Generate the alongwind force values corresponding to these locations via:

$$F_i(t) = \frac{1}{2}\rho_a C_{Di} A_i (\bar{V}_{z_i} + v_i(t_n))^2 \qquad (10)$$

   for which A_i is the exposed area, C_{Di} is the drag coefficient and ρ_a is the density of air.

(A similar approach to the above may be used for modelling the accrosswind fluctuations to thereby produce a corresponding set of accroswind force traces at selected locations along the mast).

Wind force traces generated in this way may then be used in a time history response analysis via the structural model adopted for the guyed-mast system which itself may model non-linear features of the guy-cluster system that has been chosen for the mast under consideration.

CONCLUSIONS

The modelling of the response of cable-stayed masts to the effects of high wind can be very complex.

Linear spectral based methods can be particularly expedient to apply when only the first mode is considered to dominate in the response, hence an easily implemented closed-form solution is offered in this paper for estimating the first mode natural frequency necessary to the application of these methods.

Non-linear time history response analysis methods appear to be the only methods that can reasonably account for non-linear features in the response of the taller more flexible guyed mast systems. An efficient technique for simulating wind force traces along selected positions of the mast section is summarised in this paper (that can be interfaced to such analysis methods) that takes into account the spectral and correlation properties of wind turbulence.

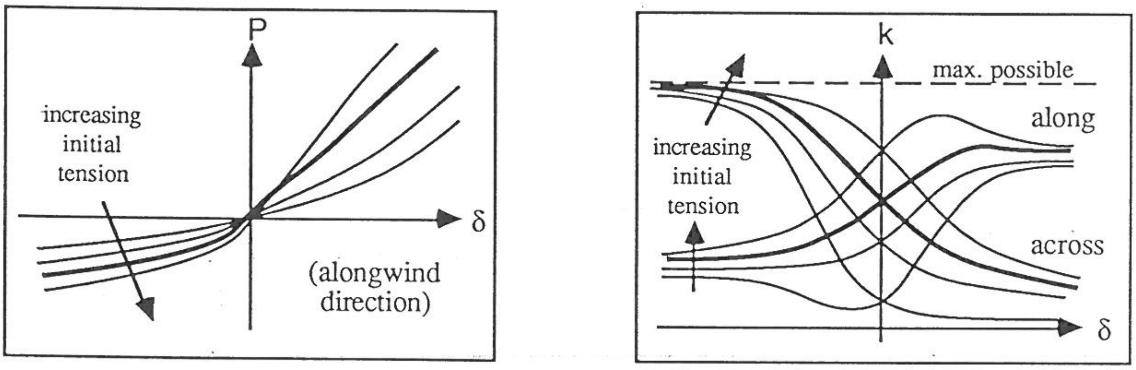

Figure1 Non-linear Horizontal Stiffness Characteristics of a 3-guy Cluster System

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