Promotion of Instability During Lock-In

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Introduction

Lock-in is the phenomenon of phase-locking of vortex shedding to the oscillatory motion of slender cylindrical structures. The process by which this takes place is poorly understood, but the phenomenon is important in the design of slender, lightly damped structures such as steel chimney stacks and towers. This is because lock-in is associated with a form of aeroelastic instability which can lead to large amplitudes of cross flow motion for these kinds of structures. The instability mechanism can be thought of as a negative-damping type; in physical terms, vortex formation becomes correlated with the motion of the structure with a relative phase such that power is transferred to the structure from the passing air-stream. Large amplitudes of motion can result if the rate at which power is transferred to the structure exceeds the rate at which internal damping forces can dissipate it. Ultimately, the amplitude of motion is limited to about ±1 diameter by non-linear aerodynamic effects.

Experimental Method

During an experimental programme conducted in the 450 kW wind tunnel at Monash University, cross flow (lift) forces which acted on short sections of a circular cylinder were measured. A feature of the equipment was that the cylinder could be forced to oscillate in a cross flow direction at amplitudes up to 3% of diameter. Since the frequency of oscillation could be varied, it was possible to vary the Reduced Velocity of oscillation ($V_r = U/fD$) while keeping the Reynolds number constant. The motion of the cylinder was measured using accelerometers installed within it.

Motion-Correlated Forces

As a part of the data reduction, lift forces which were correlated with the motion of the cylinder were calculated. Motion-correlated forces measured in smooth subcritical flow ($Re = 1.6 \times 10^5$, $I_u = 0.6\%$) showed the most interesting variation with Reduced Velocity and form the basis for discussion here. An important preliminary observation is that, even during lock-in, when the frequency of cylinder oscillation coincided with the vortex-shedding frequency, a large portion of the vortex shedding forces remained uncorrelated with motion.

The motion-correlated forces are presented here in the form of dimensionless coefficients: the Coefficient of Added Mass, C_a, and the Aerodynamic Damping Parameter K_a.

$$C_a = -C_{la} V_r^2 / 2\pi^3 \alpha$$

$$K_a = C_{lv} V_r^2 / 16\pi^2 \alpha$$

where C_{la} is the coefficient of lift correlated with cylinder acceleration and C_{lv} is the coefficient of lift correlated with cylinder velocity. The amplitude of oscillation occurs in the dimensionless form $\alpha = y_{\max}/D$. The Coefficient of Added Mass describes the acceleration-correlated forces in terms of an equivalent mass of fluid, i.e.

$$l_a = -C_a \cdot \rho \frac{\pi D^2}{4} \cdot (2\pi f)^2 \cdot \alpha D$$

where l_a is the lift force correlated with cylinder acceleration per unit length. The Aerodynamic Damping Parameter is a form of mass-damping coefficient; for a uniform cylinder moving as a rigid body, an equivalent structural damping coefficient can be computed:

$$\zeta_{\text{aero}} = -K_a \frac{\rho D^2}{m}$$

where m is the mass per unit length of the structure. This shows that a positive value of K_a corresponds to *negative* damping, that is, the structure extracts power from the flow.

Values of C_a and K_a are shown as functions of Reduced Velocity in figure 1. During the measurements, the cylinder was forced to oscillate at a nominal amplitude $\alpha = 3\%$. As can be seen, there was a rapid change in the sign of K_a with change in V_r near $V_r = 5$ (this corresponds to a Strouhal number of 0.2). This change has been observed before, and has its physical basis in the change of phase of vortex shedding with respect to cylinder motion (see for example Zdravkovich 1982). More surprising were the high values of Coefficient of Added Mass observed near synchronization ($V_r = 5$), and the negative values observed at higher V_r. Again, there are precedents for the measurements (e.g. Nakamura, Kaku & Mizota 1971), but in this case the physical basis is less clear, although it may be associated with the formation of vortices at peak values of cylinder acceleration (as reported by Williamson & Roshko 1988).

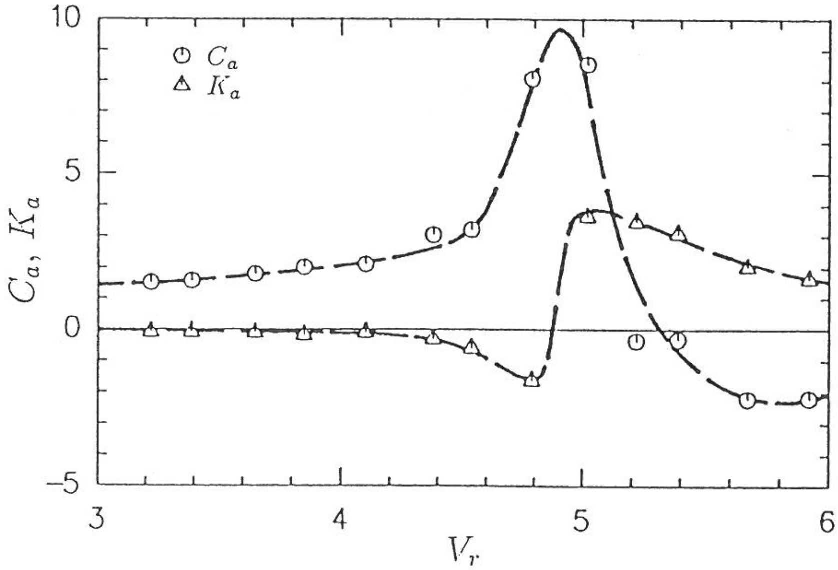

Figure 1: Values of Coefficient of Added Mass (C_a) and Aerodynamic Damping Parameter (K_a) as functions of Reduced Velocity (V_r).

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The results presented in figure 1 are replotted in polar form in figure 2, with V_r as a parameter. The asymptotic value of K_a as $V_r \to \infty$ is derived on a quasi-steady basis which gives

$K_a = -C_d V_r / 8\pi$, where C_d is the Coefficient of Drag. As $V_r \to 0$ ($f \to \infty$), it is expected (and observed) that $C_a \to 1$, the value for a cylinder oscillating at small amplitude in stationary fluid.

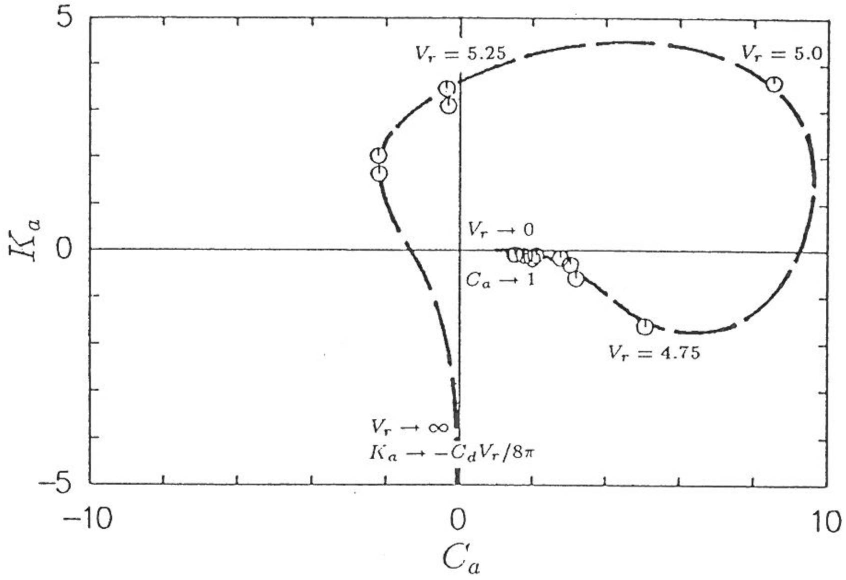

Figure 2: Values of Coefficient of Added Mass (C_a) and Aerodynamic Damping Parameter (K_a) in polar form with V_r as a parameter.

Instability, corresponding to positive values of K_a, is promoted near $V_r = 5$ by the following means. At $V_r > 5.25$, C_a is negative; this tends to increase the frequency of structural oscillation, thus decreasing the Reduced Velocity. Conversely, at $V_r < 5.25$, C_a is positive, which tends to reduce the frequency of structural oscillation, increasing V_r towards the zone of highest K_a.

To put the effect in perspective, the ratio of added mass to structural mass needs to be considered. For lightly-constructed steel chimney stacks, a typical ratio of structural mass per unit length to displaced mass of fluid per unit length is 50:1. This means that with $C_{a_{\max}} \approx 10$, as shown here, the effective structural mass per unit length is reduced by about 20%.

References

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