CALIBRATIONS OF CLOSED-FORM SOLUTIONS OF FATIGUE LIFE UNDER ALONG-WIND LOADING

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Introduction

Turbulent winds produce fluctuating loads on structures that can cause fatigue problems. Lighting columns are an example of a structure for which along-wind buffeting loading (exacerbated by Reynolds drag instability) has produced a number of incidents of premature fatigue failure at exposed sites in the U.K. (Figure 1). These failures prompted research to be undertaken at full-scale to measure the cycles of stress in 12 m high tubular steel lighting columns with respect to approach wind speed (Robertson et al, 2001), in order to appraise and develop design rules. Independently, Holmes (2001, 2002) developed a simple 'closed-form' solution for the prediction of fatigue damage to structures subjected to along-wind loading that takes account of both the resonant and background responses. Such a closed-form solution is potentially hugely attractive and valuable to engineers, but the solution needs to be validated through calibrations with real data. The full-scale testing of lighting columns at Silsoe Research Institute (SRI) generated eminently suitable data sets with which to calibrate the recently developed solution. This paper compares results obtained from the solution with those obtained from the full-scale measurements.

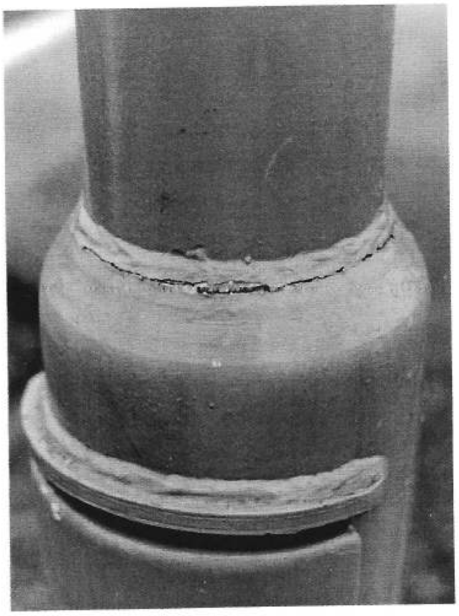

Figure 1. Fatigue crack in a lighting column in service

Full-scale data

The full-scale measurements on lighting columns have been reported (Robertson *et al*, 2001) although measurements have continued to obtain data for higher wind speeds. Tubular steel lighting columns, 12 m high with a single luminaire, were strain-gauged just above the shoulder-weld and erected at the SRI Wind Engineering site (Figure 2). Simultaneous 10-min records of bending strain in two planes, and of wind speed at 10 m height have been collected for ranges of wind speed and direction. The 100

Hz strain records (60000 data points per channel per record) were cycle-counted using 'rainflow' analysis, converted to stress, and translated into damage using a fatigue endurance (*s-N*) curve obtained from sinusoidal loading tests conducted on representative lighting columns. The data records enabled damage/hr to be plotted with respect to mean wind speed. When combined with a Weibull distribution representing the population of wind speeds to which a column will be exposed, the results enable fatigue life to be predicted.

Figure 2. Experimental lighting columns at the Silsoe test site

Closed-form solution for fatigue damage

Holmes (2002) has presented a simple closed-form solution for predicting fatigue damage to, and fatigue life of, structures subjected to along-wind loading. Expressions are presented for the narrow-band, resonant response, and for the wide-band, background response. The expression for the narrow-band fatigue damage rate is (slightly modified in terms of stress range rather than amplitude):

$$\frac{D}{T} = \frac{\lambda \nu_o^{+}}{K} \left(2\sqrt{2}\sigma\right)^m \Gamma\left(\frac{m}{2}+1\right) \qquad (1)$$

where λ is a factor equal to 1.0 for the upper bound and less than 1 for the lower bound, and ν_o^+ is the cycling rate (taken as the natural frequency of the structure, 0.89 Hz, for the upper bound, and half the natural frequency for the lower bound); K and m are obtained from the equation for the *s-N* curve (s being stress range and N the number of cycles), viz:

$$K = N s^m = N s^4 = 10^{14.137} \ (\mathrm{MPa})^4$$

Γ is the gamma function, and σ is the standard deviation of stress which is a function of the mean wind speed $\bar{U}$, given by:

$$\sigma = A \overline{U}^{\,n}$$

The constants A and n were found by evaluating the standard deviations of selected strain records and plotting these against the corresponding mean wind speeds, which gave $A = 0.152$ and $n = 1.55$. For the lower bound estimation, $\lambda = 0.794$ (Holmes, 2002). Taking logarithms of Eq. (1) and substituting the above values gives the damage rates (per second) for the lighting column as:

$$\log_e \frac{D}{T} = -35.35 + 6.2 \log_e \overline{U} \text{ (upper limit)}$$

$$\log_e \frac{D}{T} = -36.27 + 6.2 \log_e \overline{U} \text{ (lower limit)} \qquad (2)$$

Comparison of experimental and theoretical damage rates

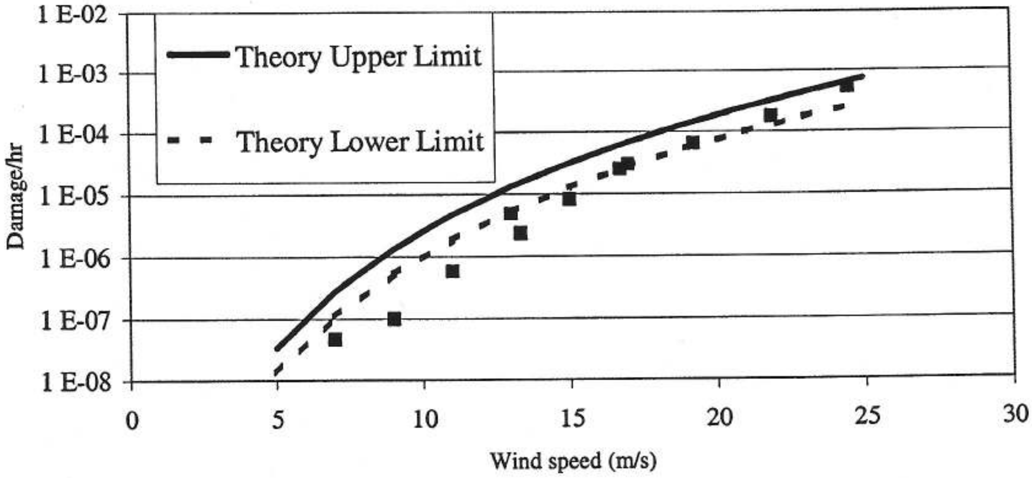

Figure 3. Comparison of directly calculated full-scale experimental fatigue damage rates and closed-form prediction

Figure 3 reveals that Eqs (2) represent reasonably good bounds to the experimental fatigue damage results at the upper end. At the lower wind speeds, the theoretical bounds are high. This is probably because of the reduced slope of the log (S) versus log(N) fatigue line at low stress ranges (Robertson et al , 2001) which was ignored in the theoretical calculations.

Holmes (2002) gave the following expressions for upper and lower limits to fatigue life :

$$T_{\text{lower}} = \frac{K}{\upsilon_o^+ (\sqrt{2}A)^m c^{mn} \Gamma(\frac{m}{2}+1)\ \Gamma(\frac{mn+k}{k})} \qquad (3)$$

$$T_{\text{upper}} = \frac{K}{\lambda \upsilon_o^+ (\sqrt{2}A)^m c^{mn} \Gamma(\frac{m}{2}+1)\ \Gamma(\frac{mn+k}{k})} \qquad (4)$$

where c and k are parameters of the Weibull probability distribution fitted to the reference mean wind speed at the site in question, in the form :

$$f_U(\overline{U}) = \frac{k\overline{U}^{k-1}}{c^k} \exp\left[-\left(\frac{\overline{U}}{c}\right)^k\right] \qquad (5)$$

The relationship between the standard deviation of stress and the mean wind speed is assumed to be given by :

$$\sigma = A\bar{U}^{n} \qquad (6)$$

For sites in the U.K., Robertson *et al* (2001) found that k is equal to1.85 and that c in Equation (5) is related to the 50-year return period hourly wind speed, $\bar{U}_{50}$ by :

$$c = \bar{U}_{50}\,(12.99)^{-1/k} \qquad (7)$$

According to the British Standard for towers and masts, (BSI.1995), for the test site at Silsoe, $\bar{U}_{50}$ is 24.9m/s, giving c equal to 6.23 m/s. For exposed sites in the north of England, where premature fatigue has been experienced in these lighting columns, $\bar{U}_{50}$ is about 32.5 m/s; Equation (6) then gives c equal to 8.13 m/s.

Substitution into Equations (3) and (4), then gives the upper and lower estimates of fatigue life in Table I.

Table I. Estimates of fatigue life

Site	$\bar{U}_{50}$ (m/s)	c (m/s)	T_{lower} (years)	T_{upper} (years)
N. England exposed	32.5	8.13	17	27
Silsoe	24.9	6.23	90	142

The estimated life of about twenty years at exposed sites in the North of England corresponds generally to the experience for lighting columns located at such sites, although heavier columns would usually be used there based on static wind load calculations. A much greater life of around 100 years is predicted for columns at the Silsoe site.

Conclusions

Recent experimental studies of stress-wind speed relationships under along-wind loading of lighting columns, and fatigue 'cycle' counts from actual time histories of stress, has enabled 'calibration' of a theoretical model of fatigue damage rate and fatigue life to be carried out. Theoretical damage rates as a function of mean wind speed agree well with rates directly calculated from the experimental data. Theoretical fatigue life estimates also agree quite well with experience on lighting columns in service in the North of England.

References

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