MODE SHAPE CORRECTIONS FOR SWAY AND TWIST MODES

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Introduction

The high-frequency base balance (HFBB) is now more than twenty years old (Tschanz, 1982), and has become the standard wind-tunnel method by which overall structural wind loads, and responses such as accelerations, displacements and velocities, are determined for tall buildings at the design stage. Essentially the mean (time-averaged) and quasi-static background base bending moments and torques are determined by direct measurement, but the resonant dynamic components are computed from the recorded time histories or spectral densities of the base moments. The simplicity of the wind-tunnel models, the rapidity at which tests can be carried out, and the ease by which changes in basic dynamic properties such a frequency and damping can be incorporated, greatly outweigh the disadvantage of the neglect of aeroelastic effects, generally regarded as negligible for the majority of habitable tall buildings. In addition, the HFBB approach allows for inclusion of contributions from the fundamental twist (torsional) mode, and mode shape corrections to the sway modes, normally ignored in simple aeroelastic models

Although this is a well-established technique in both North America and Australia, there are still significant differences in the methods used to correct the measured spectral densities of base bending moments and torques to give the required spectral densities of generalized forces for sway and twist modes, subsequently used to compute the resonant dynamic response of the building. This is particularly true for the twist modes which have a mode shape increasing monotonically upwards from zero at ground level; on the other hand the base torque measured by an HFBB is derived from the local torque at each height level *uniformly* weighted with height. The mode shape corrections, in this case, are therefore quite significant, but also vary greatly between various wind-tunnel laboratories.

In this paper, mode shape corrections for both sway and twist modes are reviewed, compared with experimental data, and recommended approaches are given. Note that corrections are only required for the resonant components of base bending moments and torque. The mean and background components can be obtained directly from the output of the HFBB.

Correction for Sway Modes

In the following, it will be assumed that mode shape can be fitted by a power function of the form

$$\mu(z) = \left(\frac{z}{h}\right)^{\beta} \qquad (1)$$

A number of authors have considered the theoretical corrections required to corrected the spectra of linearly weighted base bending moments to those for generalized forces. Holmes (1987) derived the following correction factor assuming low correlation between the fluctuating sectional forces at any pair of height levels on the building. The spectral density of fluctuating sectional forces were assumed to be invariant with height.

$$S_{Fx}(n) = \left(\frac{1}{h}\right)^{2}\left(\frac{3}{1+2\beta}\right)S_{My}(n) \qquad (2)$$

Holmes (1987) also derived the corresponding limit for full correlation of the fluctuating sectional forces, and proposed the following as an intermediate correction factor between the low and high correlation limits.

$$S_{Fx}(n) = \left(\frac{1}{h}\right)^2 \left(\frac{4}{1+3\beta}\right) S_{My}(n) \qquad (3)$$

Boggs and Peterka (1989) only considered the full correlation limit, and assumed that the fluctuating forces varied with height as a power law. Vickery *et al.* (1985) made some direct measurements of the mode shape correction, using time histories of sectional forces based on pressure measurements. Xu and Kwok (1993) modified the low and high limits of Holmes (1987) to account for other variations of spectral density with height.

The following table summarizes the theoretical correction factors for various values of the mode shape exponent β, and of the exponent, γ, used to describe the variation of spectral density of fluctuating sectional forces with height in the form :

$$S_f(n, z) = S_f(n)_{max} \cdot (z/h)^{2\gamma} \qquad (4)$$

Table I. Correction factors for spectral densities of generalized forces in sway modes

β	Low correlation $\frac{3+2\gamma}{1+2\gamma+2\beta}$			High correlation $\left(\frac{2+\gamma}{1+\gamma+\beta}\right)^2$			Recommended $\frac{4}{1+3\beta}$	Measured* Vickery *et al*
	γ = 0	γ = 0.25	γ = 0.5	γ = 0	γ = 0.25	γ = 0.5		
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	-
1.25	0.86	0.88	0.89	0.79	0.81	0.83	0.84	0.86
1.5	0.75	0.78	0.80	0.64	0.67	0.69	0.73	0.71
2.0	0.60	0.64	0.67	0.44	0.48	0.51	0.57	0.61

The measurements of Vickery *et al* were averages over the reduced frequency range of interest for the resonant response of tall buildings in urban exposures. For the mode shape exponents in the table, which cover the typical range for tall buildings, the measured factors generally fall in between the high and low correlation limits, as expected. Note that assuming high correlation is unconservative for β > 1.0.

For practical purposes a simple function *which does not require knowledge of the variation of spectral density with height* is desirable. This is provided by Equation (3), i.e. the function $\left(\frac{4}{1+3\beta}\right)$, suggested by Holmes (1987). As shown in Table I, this fits the experimental data of Vickery *et al* well. Another alternative would be the slightly more conservative low correlation limit for γ = 0, Equation (2), which also matches the experimental data well.

To determine the acceleration in this mode, the spectral density of the generalized force is factored by the transfer function for generalized acceleration. This involves the square of the generalized mass in the denominator. This is the mass per unit height each height level multiplied by the square of the mode shape, and integrated over the height of the structure.Then the effective correction factor for mean square angular acceleration (to correct from the assumption of a linear mode shape, β =0) is:

$$\eta_a^{\;2} = \left(\frac{4}{1+3\beta}\right)\left(\frac{1+2\beta}{3}\right)^2 \qquad (5)$$

For example, if β is equal to 1.5, the above factor is equal to (0.727)(1.78) = 1.29, or a factor of 1.14 on the standard deviation and peak accelerations. Usually, the second term is implicitly included by using a generalized mass computed from the actual mode shape for the building, rather than a power law fit.

To determine the mean square resonant base moment, the moment due to the inertial forces arising from mass times acceleration at each height level is calculated. The resulting mode shape correction factor to the mean square resonant base bending moment is then :

$$\eta_M{}^2 = \left(\frac{4}{1+3\beta}\right)\left(\frac{1+2\beta}{2+\beta}\right)^2 \qquad (6)$$

For a value of β of 1.5, the resultant correction factor for the mean square resonant base moment is 0.95. As the resonant component is usually about one half the total peak base bending moment the total error in neglecting mode shape corrections is typically only about 1-2%. Corrections to base bending moments are commonly ignored.

Correction for Twist Modes

As for the sway modes, it will be assumed that mode shape can be fitted by a power function of the form:

$$\mu_t(z) = \left(\frac{z}{h}\right)^{\beta_t} \qquad (7)$$

As for the sway modes, a power law variation of the spectral density of fluctuating sectional torque with height can be assumed :

$$S_t(n, z) = S_t(n)_{max} \cdot (z/h)^{2\gamma} \qquad (8)$$

Based on a uniform distribution (γ=0), and low correlation, the following correction factor, is obtained :

$$S_{Ft}(n) = \left(\frac{1}{1+2\beta_t}\right) S_{Mz}(n) \qquad (9)$$

Alternative corrections derived by Boggs and Peterka (1989) allowed for variation of the fluctuating torque with height, but assumed full correlation of these fluctuating sectional torques over height separations. The theoretical correction factors obtained with assumptions of both high and low correlation, are shown in Table II, together with the analysis of Tallin and Ellingwood (1985), based on measurements of Reinhold.

The table shows that a correction factor based on low correlation and uniform fluctuating torques with height (Equation (9)) fits the data analysed by Tallin and Ellingwood (1985) well. This is not surprising as the correlations between fluctuating torques at various heights were computed by Tallin and Ellingwood, and were found to be very low. On the other hand the commonly used correction factor of $0.49 = (0.7)^2$ is over-conservative.

Table II. Correction factors for spectral densities of generalized forces in twist modes

β_t	Low correlation $\frac{1+2\gamma}{1+2\gamma+2\beta_t}$			High correlation $\left(\frac{1+\gamma}{1+\gamma+\beta_t}\right)^2$			Recommended $\frac{1}{1+2\beta_t}$	Measured Tallin & Ellingwood
	$\gamma=0$	$\gamma=0.25$	$\gamma=0.5$	$\gamma=0$	$\gamma=0.25$	$\gamma=0.5$		
0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	-
0.75	0.40	0.50	0.57	0.33	0.39	0.44	0.40	-
1.0	0.33	0.43	0.50	0.25	0.31	0.36	0.33	0.32
1.25	0.29	0.38	0.44	0.20	0.25	0.30	0.29	-
1.5	0.25	0.33	0.40	0.16	0.21	0.25	0.25	0.26

The correction factors for angular acceleration and resonant base torque are obtained in a similar way to the sway modes, giving Equations (10) and (11) respectively.

$$\eta_{at}^{\;2} = \left(\frac{1}{1+2\beta_t}\right)\left(\frac{1+2\beta_t}{1}\right)^2 \qquad (10)$$

$$\eta_T^{\;2} = \left(\frac{1}{1+2\beta_t}\right)\left(\frac{1+2\beta_t}{1+\beta_t}\right)^2 \qquad (11)$$

For β_t = 1.0, the correction factor to angular acceleration is thus $(1/3)(3/1)^2 = 3$, and the correction factor to the standard deviation acceleration is the square root of this, i.e. 1.73. As for the sway modes, usually the procedure is to calculate the generalized mass from the actual mode shape, and apply that part of the correction factor implicitly. For β_t = 1.0, the correction factor to the mean square resonant base torque is thus $(1/3)(3/2)^2 = 0.75$, and the correction factor to the standard deviation resonant base torque is the square root of this, i.e. 0.87.

Conclusions

Correction factors to the outputs of a HFBB to give the spectral densities of generalized forces in sway modes and the twist mode are proposed. These are respectively Equations (3) and (9). They have been validated by comparisons with direct measurements of these factors.

References

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