Examination of the Six Aerodynamic Admittance Functions of Bridge Decks

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Abstract The Aerodynamic Admittance Function (AAF) is one of the most important aerodynamic parameters for long-span cable-stayed or suspension bridges to predict the turbulence-induced buffeting responses. In this paper, four estimators of the six AAFs were theoretically deduced by techniques generally adopted in the field of system identification, which might further enhance the interpretation and the meaning of AAFs. Wind tunnel tests were conducted to determine the six AAFs by the proposed estimators and to illustrate the significance of the six AAFs. The effectiveness of the proposed estimators was further tested by the predicted forces and pitching moment spectra.

1. INTRODUCTION

For long-span, flexible cable-stayed or suspension bridges, the gust-induced buffeting forces and responses must be accurately predicted.

In the process of turbulence buffeting, AAFs define the effectiveness of different wind gust sizes and frequencies in generating spatially well-correlated wind forces or moments on a bridge deck. From the point of system identification, the aerodynamic admittance functions actually serve as the transfer functions between the fluctuations of wind velocity and the random buffeting forces.

Six AAFs exist (Scanlan, 2001) that relate longitudinal (U) and vertical (W) components of wind velocity to the buffeting forces and moment. Since it is difficult to separate the contributions of the U- and W- components of wind velocity, the AAFs corresponding to the lift force, drag force and the pitching moment are generally lumped into a single AAF, and the number of AAFs thus reduced to three (Larose, 1996).

In this paper, efforts were made to extract the six AAFs by the four generally accepted estimators of the Frequency Response Functions (FRF) used in the field of system identification (Fu, 2002). Wind tunnel tests were conducted to determine the six AAFs by the proposed estimators and to get some general information about the six AAFs, especially those corresponding to the U- component.

2. THEORETICAL FORMULATION

For a system with three Degrees-Of-Freedom (DOF), e.g. L_b , D_b and M_b , the frequency domain responses of the system to external forces U and W can be expressed as,

$$\begin{bmatrix} L_b(\omega) \\ D_b(\omega) \\ M_b(\omega) \end{bmatrix} = \begin{bmatrix} \chi^0_{Lu}(\omega) & \chi^0_{Lw}(\omega) \\ \chi^0_{Du}(\omega) & \chi^0_{Dw}(\omega) \\ \chi^0_{Mu}(\omega) & \chi^0_{Mw}(\omega) \end{bmatrix} \begin{bmatrix} U(\omega) \\ W(\omega) \end{bmatrix} \quad (1)$$

where: χ^0_{ij}, ($i = L, D, M$ and $j = u, w$), is the FRF measuring the potential of the system to magnify the input force and ω denotes frequency.

Right multiplying by the complex conjugate transpose of the input forces, Eq. (1) can be rewritten as,

$$\begin{bmatrix} L_b \\ D_b \\ M_b \end{bmatrix} \begin{bmatrix} U \\ W \end{bmatrix}^H = \begin{bmatrix} \chi^0_{Lu} & \chi^0_{Lw} \\ \chi^0_{Du} & \chi^0_{Dw} \\ \chi^0_{Mu} & \chi^0_{Mw} \end{bmatrix} \begin{bmatrix} U \\ W \end{bmatrix} \begin{bmatrix} U \\ W \end{bmatrix}^H \quad (2)$$

According to the estimators (Bloomfield, 1976) of the auto- and cross- Power Spectral Densities (PSD), Eq. (2) can be further simplified as:

$$\begin{bmatrix} S_{uL} & S_{wL} \\ S_{uD} & S_{wD} \\ S_{uM} & S_{wM} \end{bmatrix} = \begin{bmatrix} \chi^0_{Lu} & \chi^0_{Lw} \\ \chi^0_{Du} & \chi^0_{Dw} \\ \chi^0_{Mu} & \chi^0_{Mw} \end{bmatrix} \begin{bmatrix} S_{uu} & S_{wu} \\ S_{uw} & S_{ww} \end{bmatrix} \quad (3)$$

Therefore,

$$\begin{bmatrix} \chi^0_{Lu} & \chi^0_{Lw} \\ \chi^0_{Du} & \chi^0_{Dw} \\ \chi^0_{Mu} & \chi^0_{Mw} \end{bmatrix} = \begin{bmatrix} S_{uL} & S_{wL} \\ S_{uD} & S_{wD} \\ S_{uM} & S_{wM} \end{bmatrix} \begin{bmatrix} S_{uu} & S_{wu} \\ S_{uw} & S_{ww} \end{bmatrix}^{-1} \quad (4)$$

Eq.(4) is sometimes called the $\hat{H}_1$ estimator of FRF, which estimates FRF by using the cross input-output spectra and the input auto-spectra.

Alternatively, right-multiplying by the complex conjugate transpose of the output responses, instead

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of input forces, the $\hat{H}_2$ estimator of FRF can be obtained:

$$\begin{bmatrix} \chi^0_{Lu} & \chi^0_{Lw} \\ \chi^0_{Du} & \chi^0_{Dw} \\ \chi^0_{Mu} & \chi^0_{Mw} \end{bmatrix} = \begin{bmatrix} S_{LL} & S_{DL} & S_{ML} \\ S_{LD} & S_{DD} & S_{MD} \\ S_{LM} & S_{ML} & S_{MM} \end{bmatrix} \begin{bmatrix} S_{Lu} & S_{Du} & S_{Mu} \\ S_{Lw} & S_{Dw} & S_{Mw} \end{bmatrix}^+ \tag{5}$$

where the superscript $^+$ denotes the pseudo-inverse. In other words, the $\hat{H}_2$ estimator of FRF is obtained by normalizing the output auto-spectra by the cross input-output spectra.

Since the $\hat{H}_1$ estimator always underestimates FRF, but the $\hat{H}_2$ estimator always overestimates it (Fu, 2002 and Maia, 1997), different averages of these two estimators were introduced to improve the quality of the estimated FRF:

- Algebraic Average (the $\hat{H}_3$ estimator of FRF):

$$\hat{H}_3 = \frac{\hat{H}_1 + \hat{H}_2}{2} \tag{6}$$

- Geometric Average (the $\hat{H}_4$ estimator of FRF):

$$\hat{H}_4 = \sqrt{\hat{H}_1 \hat{H}_2} \tag{7}$$

In the field of bridge aerodynamics, the sectional buffeting lift force L_b, drag force D_b and pitching moment M_b, of bridge decks have conventionally been written in quasi-static terms (Scanlan, 2001):

$$\begin{aligned} L_b(\omega) &= \left(\frac{1}{2}\rho\bar{u}^2 B\right)\left[2C_L \frac{U(\omega)}{\bar{u}}\chi_{Lu}(\omega) + \left(C_L' + C_D\right)\frac{W(\omega)}{\bar{u}}\chi_{Lw}(\omega)\right] \\ D_b(\omega) &= \left(\frac{1}{2}\rho\bar{u}^2 B\right)\left[2C_D \frac{U(\omega)}{\bar{u}}\chi_{Du}(\omega) + \left(C_D'\right)\frac{W(\omega)}{\bar{u}}\chi_{Dw}(\omega)\right] \\ M_b(\omega) &= \left(\frac{1}{2}\rho\bar{u}^2 B^2\right)\left[2C_M \frac{U(\omega)}{\bar{u}}\chi_{Mu}(\omega) + \left(C_M'\right)\frac{W(\omega)}{\bar{u}}\chi_{Mw}(\omega)\right] \end{aligned} \tag{8}$$

in which:

ρ = air mass density; $\bar{u}$ = mean wind velocity; B = deck width; C_L, C_D, C_M = static lift, drag and moment coefficients; C_L', C_D', C_M' = the corresponding slope of the force or moment coefficients with respect to wind angle of attack; $U(\omega), W(\omega)$ = Fourier Transform of the U- and W-components of the fluctuations of wind velocity; χ_{ij} = ($i = L, D, M$ and $j = u, w$) Aerodynamic Admittance Function.

Eq.(8) can be rewritten as a matrix equation similar to Eq.(1) and the AAFs can thus be estimated by the aforementioned $\hat{H}_1$, $\hat{H}_2$, $\hat{H}_3$ and $\hat{H}_4$ estimators, supposing:

$$\begin{aligned} \chi^0_{Lu}(\omega) &= \frac{1}{2}\rho\bar{u}B \times 2C_L \times \chi_{Lu}(\omega) \\ \chi^0_{Lw}(\omega) &= \frac{1}{2}\rho\bar{u}B \times \left(C_L' + C_D\right) \times \chi_{Lw}(\omega) \\ \chi^0_{Du}(\omega) &= \frac{1}{2}\rho\bar{u}B \times 2C_D \times \chi_{Du}(\omega) \\ \chi^0_{Dw}(\omega) &= \frac{1}{2}\rho\bar{u}B \times C_D' \times \chi_{Dw}(\omega) \\ \chi^0_{Mu}(\omega) &= \frac{1}{2}\rho\bar{u}B^2 \times 2C_M \times \chi_{Mu}(\omega) \\ \chi^0_{Mw}(\omega) &= \frac{1}{2}\rho\bar{u}B^2 \times C_M' \times \chi_{Mw}(\omega) \end{aligned} \tag{9}$$

The existence of the analytical form of the inverse matrix in Eq.(4) allows that equation to be explicitly expressed as:

$$\begin{bmatrix} \chi^0_{Lu} & \chi^0_{Lw} \\ \chi^0_{Du} & \chi^0_{Dw} \\ \chi^0_{Mu} & \chi^0_{Mw} \end{bmatrix} = \frac{\begin{bmatrix} S_{uL} & S_{wL} \\ S_{uD} & S_{wD} \\ S_{uM} & S_{wM} \end{bmatrix}\begin{bmatrix} S_{ww} & -S_{wu} \\ -S_{uw} & S_{uu} \end{bmatrix}}{S_{uu}S_{ww} - S_{uw}S_{wu}} \tag{10}$$

Therefore, the $\hat{H}_1$ estimator of the six AAFs can be explicitly expressed as:

$$\begin{aligned} \chi_{Lu}(\omega) &= \frac{S_{uL}(\omega)S_{ww}(\omega) - S_{wL}(\omega)S_{uw}(\omega)}{\frac{1}{2}\rho\bar{u}B \times 2C_L \times [S_{uu}(\omega)S_{ww}(\omega) - S_{uw}(\omega)S_{wu}(\omega)]} \\ \chi_{Lw}(\omega) &= \frac{S_{wL}(\omega)S_{uu}(\omega) - S_{uL}(\omega)S_{wu}(\omega)}{\frac{1}{2}\rho\bar{u}B \times \left(C_L' + C_D\right) \times [S_{uu}(\omega)S_{ww}(\omega) - S_{uw}(\omega)S_{wu}(\omega)]} \\ \chi_{Du}(\omega) &= \frac{S_{uD}(\omega)S_{ww}(\omega) - S_{wD}(\omega)S_{uw}(\omega)}{\frac{1}{2}\rho\bar{u}B \times 2C_D \times [S_{uu}(\omega)S_{ww}(\omega) - S_{uw}(\omega)S_{wu}(\omega)]} \\ \chi_{Dw}(\omega) &= \frac{S_{wD}(\omega)S_{uu}(\omega) - S_{uD}(\omega)S_{wu}(\omega)}{\frac{1}{2}\rho\bar{u}B \times \left(C_D'\right) \times [S_{uu}(\omega)S_{ww}(\omega) - S_{uw}(\omega)S_{wu}(\omega)]} \\ \chi_{Mu}(\omega) &= \frac{S_{uM}(\omega)S_{ww}(\omega) - S_{wM}(\omega)S_{uw}(\omega)}{\frac{1}{2}\rho\bar{u}B^2 \times 2C_M \times [S_{uu}(\omega)S_{ww}(\omega) - S_{uw}(\omega)S_{wu}(\omega)]} \\ \chi_{Mw}(\omega) &= \frac{S_{wM}(\omega)S_{uu}(\omega) - S_{uM}(\omega)S_{wu}(\omega)}{\frac{1}{2}\rho\bar{u}B^2 \times \left(C_M'\right) \times [S_{uu}(\omega)S_{ww}(\omega) - S_{uw}(\omega)S_{wu}(\omega)]} \end{aligned} \tag{11}$$

Obviously, all four proposed estimators of the six AAFs will contain complex numbers, i.e. both the amplitude and phase information of the six AAFs will be available.

3. WIND TUNNEL TEST INVESTIGATION

A rigid pressure sectional model of a bridge (53.3m wide by 3.5m deep) was tested to estimate the six AAFs by the aforementioned estimators. Wind tunnel tests were conducted in the low turbulence, 3m by 2m high-speed working section of the CLP Power Wind/Wave Tunnel Facility at the Hong Kong University of Science and Technology. Grids placed upstream of the bridge model were used to generate turbulent flow with about 17% (longitudinal) and 15% (vertical) turbulence intensities and 35m (longitudinal) and 17m (vertical) (prototype scale) turbulence length scales.

Time histories of the buffeting lift force, drag force and pitching moment were obtained by integrating the pressures measured on the pressure-tapped strip, while time histories of different components of wind velocities were measured by a Turbulent Flow Instrumentation (TFI) Cobra Probe. Each of the time histories was approximately 135 seconds, equivalent to around one hour at prototype scale, and was sampled at a rate of 400Hz.

Based on the auto- and cross spectral analysis of the time histories, the six AAFs were then calculated by the aforementioned estimators. Figure 1 shows the $\hat{H}_1$, $\hat{H}_2$, $\hat{H}_3$ and $\hat{H}_4$ estimators of $|\chi_{Lu}(\omega)|^2$ and $|\chi_{Lw}(\omega)|^2$, as well as the auto- PSD of lift force predicted by:

$$S_{LL}(\omega)=\left(\frac{1}{2}\rho\bar{u}^2B\right)^2\left[4C_L^2\frac{S_{uu}(\omega)}{\bar{u}^2}|\chi_{Lu}(\omega)|^2+\left(C_L'+C_D\right)^2\frac{S_{ww}(\omega)}{\bar{u}^2}|\chi_{Lw}(\omega)|^2\right] \tag{12}$$

It can be observed from Figure 1 that the four estimators of the six AAFs satisfy the following inequality:

$$\left|\hat{H}_1\right|\le\left|\hat{H}_4\right|\le\left|\hat{H}_3\right|\le\left|\hat{H}_2\right| \tag{13}$$

which also holds for the corresponding predicted lift forces. Similar results were observed for the other four AAFs, the drag force and the pitching moment.

The $\hat{H}_4$ estimator of the six AAFs is presented in Figure 2, where it is compared to Liepmann's approximation of Sears' function and the three simplified approximations:

$$\begin{aligned}
|\chi_L(\omega)|^2 &= \frac{\bar{u}^2S_{LL}(\omega)}{\left(\frac{1}{2}\rho\bar{u}^2B\right)^2\left[4C_L^2S_{uu}(\omega)+\left(C_L'+C_D\right)^2S_{ww}(\omega)\right]}\\
|\chi_D(\omega)|^2 &= \frac{\bar{u}^2S_{DD}(\omega)}{\left(\frac{1}{2}\rho\bar{u}^2B\right)^2\left[4C_D^2S_{uu}(\omega)+\left(C_D'\right)^2S_{ww}(\omega)\right]}\\
|\chi_M(\omega)|^2 &= \frac{\bar{u}^2S_{MM}(\omega)}{\left(\frac{1}{2}\rho\bar{u}^2B^2\right)^2\left[4C_M^2S_{uu}(\omega)+\left(C_M'\right)^2S_{ww}(\omega)\right]}
\end{aligned} \tag{14}$$

It can be observed from Figure 2, that for the bridge sectional model adopted in wind tunnel tests, the following three inequalities hold:

$$\begin{aligned}
|\chi_{Lw}(\omega)|^2 &\le |\chi_L(\omega)|^2 \le |\chi_{Lu}(\omega)|^2\\
|\chi_{Du}(\omega)|^2 &\le |\chi_D(\omega)|^2 \le |\chi_{Dw}(\omega)|^2\\
|\chi_{Mw}(\omega)|^2 &\le |\chi_M(\omega)|^2 \le |\chi_{Mu}(\omega)|^2
\end{aligned} \tag{15}$$

It is also apparent that the three AAFs in each of the above inequalities have similar forms.

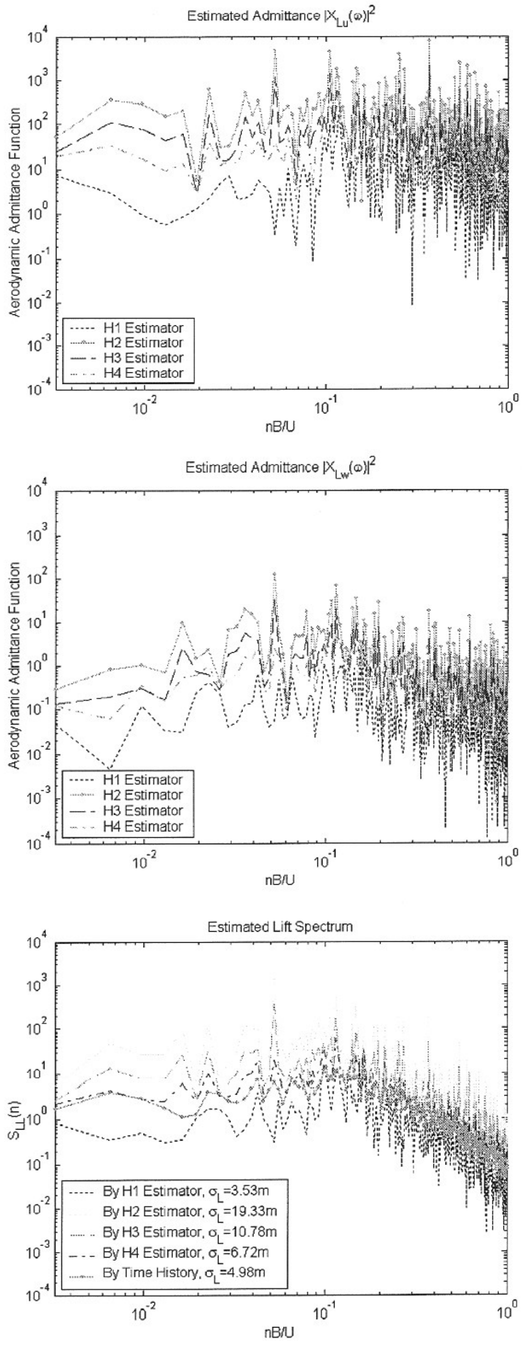

Figure 1 The two AAFs related to lift force and the predicted lift spectrum

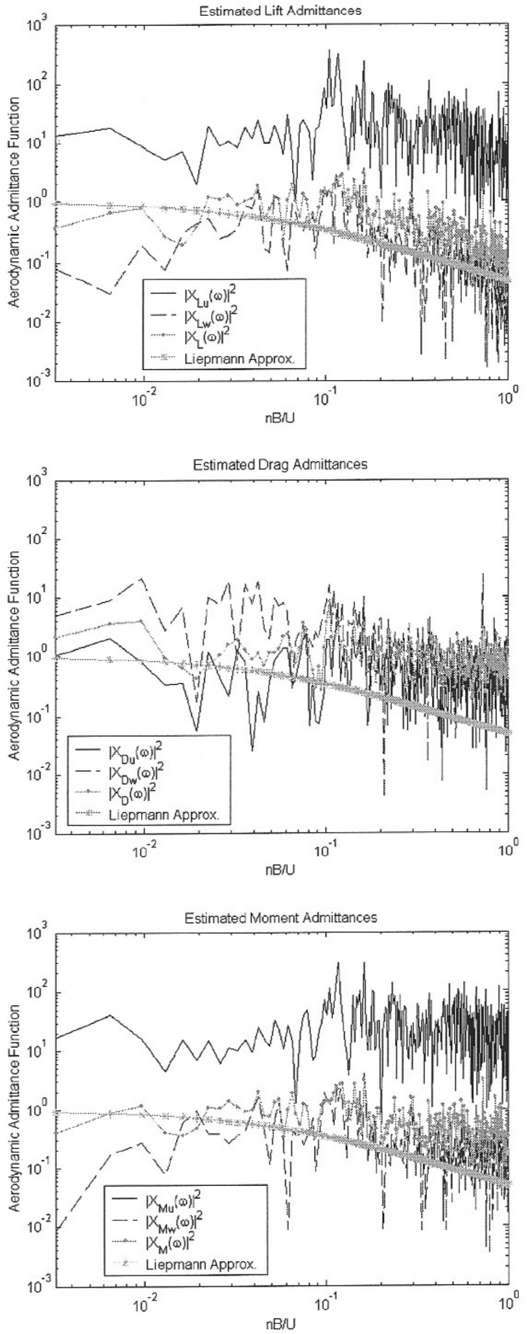

Figure 2 The $\hat{H}_4$ estimators of the six AAFs in comparison with Liepmann's approximation of Sears function and the three simplified approximations of the

4. CONCLUDING REMARKS

Four estimators, $\hat{H}_1$, $\hat{H}_2$, $\hat{H}_3$ and $\hat{H}_4$, of the six Aerodynamic Admittance Functions (AAF) for bridge decks were proposed, based on system identification techniques. Wind tunnel pressure tests of a bridge deck were conducted in the high-speed section of the CLP Wind/Wave Tunnel Facility at the Hong Kong University of Science and Technology to determine the six AAFs using the proposed estimators.

From the proposed estimators, the lower and upper limits of the six AAFs were obtained, and both the amplitude and phase information of the AAFs are available.

The lift and moment AAFs of the tested model related to the U-component of wind velocity are greater than those related to the W-component. Of the drag AAFs, the one related to the U-component has smaller magnitude.

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REFERENCE

[1] Bloomfield, P.(1976). *Fourier Analysis of Time Series: An Introduction*. New York: Wiley.

[2] Chen, X.Z., Matsumoto, M. and Kareem, A. (2000). *Aerodynamic Coupling Effects of Flutter And Buffeting of Bridges*. Journal of Engineering Mechanics, ASCE, 126(1), 17-26.

[3] Fu, Z.F and Hua, H.X. (2002). *Modal Analysis: Theory and Application* (in Chinese). Shanghai: Press of Shanghai Jiao Tong University.

[4] Larose, G.L. (1996). *Direct Measurements of Buffeting Wind Forces on Bridge Decks*. Journal of Wind Engineering and Industrial Aerodynamics, 74-76(1996), 809-818.

[5] Maia, N.M.M. and Silva, J.M.M. (1997). *Theoretical and Experimental Modal Analysis*, New York: Wiley. Research Studies Press Ltd. and John Wiley & Sons Inc.

[6] Scanlan, R.H. (2001). *Reexamination of Sectional Aerodynamic Force Functions for Bridges*. Journal of Wind Engineering and Industrial Aerodynamics, 89(2001), 1257-1266.

[7] Stoyanoff, S. (2001). *A Uniformed Approach for 3D Stability and Time Domain Response Analysis with Application of Quasi-Steady Theory*. Journal of Wind Engineering and Industrial Aerodynamics, 89(2001), 1591-1606.