# Estimation of the Across-Wind Fluctuating Moment and Spectral Density Coefficient of Rectangular Tall Buildings with Various Side Ratios

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## Abstract

This study makes an attempt to propose an approximate equation of the across-wind fluctuating moment and spectral density coefficient as a function of the side ratios of buildings. In order to estimate empirical formula, the wind tunnel tests were conducted on the aeroelastic model of the rectangular prisms with various breadth-to-depth ratios in turbulent boundary layer flows. The eleven types of models, with aspect ratio $H/\sqrt{BD}=6$ and side ratios $D/B=1/5$ to $5/1$, are investigated.

## 1. Introduction

The analytical methods for along-wind response, which is called gust factor approach or spectral modal analysis, are applicable to most prismatic structures [1] [2]. The main reason why these are possible is that the along-wind response of most structures originates almost entirely from the action of wind gustiness of the longitudinal component of the wind velocity. The power spectral density of longitudinal wind velocity has wide-band shape and dose not significantly change its shape according to the conditions of terrain, the height above ground and wind speed. Therefore, the analytical methods, using spectral and spatial correlation considerations to predict the along-wind response, have become highly developed and are included in a number of wind loading codes.

By comparison, the across-wind forcing mechanism has proved to be so complex that as yet there is no generalized analytical method available to calculate across-wind response of structures. Because, the across-wind power spectral density is significantly changed by aspect ratios, side ratios, conditions of terrains, wind speeds and dynamic characteristics of structures. In special, the vortex shedding occurred to the rear of building is strongly affected by the side ratios of buildings. In these reason, it is very difficult for the across-wind force spectral density to generalize empirical equation. A lot of wind tunnel test for the vortex induced vibration have been carried out for rectangular prisms and various interesting features of the vortex induced vibration have already been found [3] [4] [5]. However, the attempt to introduce an empirical equation of the across-wind force spectral density is rare

This study aims at proposing an empirical equation of the across-wind fluctuating moment and spectral density coefficient as a function of the side ratios of buildings. In order to estimate empirical formula, wind tunnel tests have been conducted on aeroelastic model of rectangular prisms with various breadth-to-depth ratios in turbulent boundary layer flows. In this paper, criteria of across-wind fluctuating force and spectra density are briefly discussed and then these results were mainly analyzed as a function of the side ratios of buildings. Finally, approximate equation for the across-wind fluctuating moment and spectral density coefficient were proposed.

## 2. The model and measurement

### 2.1 The wind model

The wind tunnel tests for across-wind force were performed in the boundary layer wind tunnel in Kumoh National Institute of Technology in Korea. The working section of the tunnel is $1.54m \times 1.3m$ and $14m$ long. The tests were carried out under the turbulent boundary flows over suburban areas. In order to simulate the boundary layer flow, the spires were set at the entrance of the working section and the roughness blocks were placed on the tunnel floor. The characteristics of the $1/400$ scale approach flow boundary layer wind model are given in Fig.1. The mean velocity profiles in the turbulent boundary layer flow agree well with the power law of which exponent is $0.22$ for suburban areas categorized in Korean Wind Loading Code 2000.

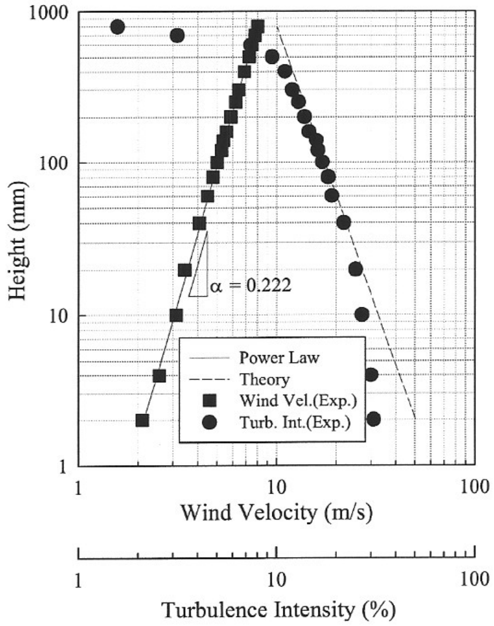


**Fig.1 Profiles of mean velocities and longitudinal turbulence intensities**

### 2.2 The aeroelastic model

The model scale is $1/400$. It was decided from the comparison of turbulent scale in the wind tunnel with the natural wind. The building models were constructed by the balsa with light weight and strong stiffness. Therefore, these are considered as a linear mode aeroelastic models. The model shapes have rectangular section. For all of the modes, the aspect ratio $H/\sqrt{BD}$, where $H$ is the height, $D$ is the depth and $B$ is the breadth of building, was fixed at $6.0$. But, the side ratios $D/B$ were varied 11 types, that is $1/5,\ 1/4,\ 1/3,\ 1/2,\ 1/1.5,\ 1/1,\ 1.5/1,\ 2/1,\ 3/1,\ 4/1$ and $5/1$.

### 2.3 Measurement

Eleven types of building models with a rectangular section were mounted on the HFFB (High Frequency Force Balance). Model and HFFB were connected strongly with together at the base. The natural frequency of this system was indicated about $150Hz$. This is enough high frequency, so that resonant problem may not be occurred in this system and can be measured data properly. The dynamic wind forces for building models were measured through HFFB.

The conditions for measuring the dynamic wind force are briefly as follows; model length scale is $1/400$; the wind velocity ratio is $1/5$; the time ratio is $1/80$; the sampling frequency is $200Hz$; the sampling is 8th round; and the low pass filter cutting frequency is $100Hz$.

## 3. Experimental results and discussion

### 3.1 Parameters and configurations

The across-wind force data are presented as mean, standard deviation force coefficient defined along with other parameters as follows:

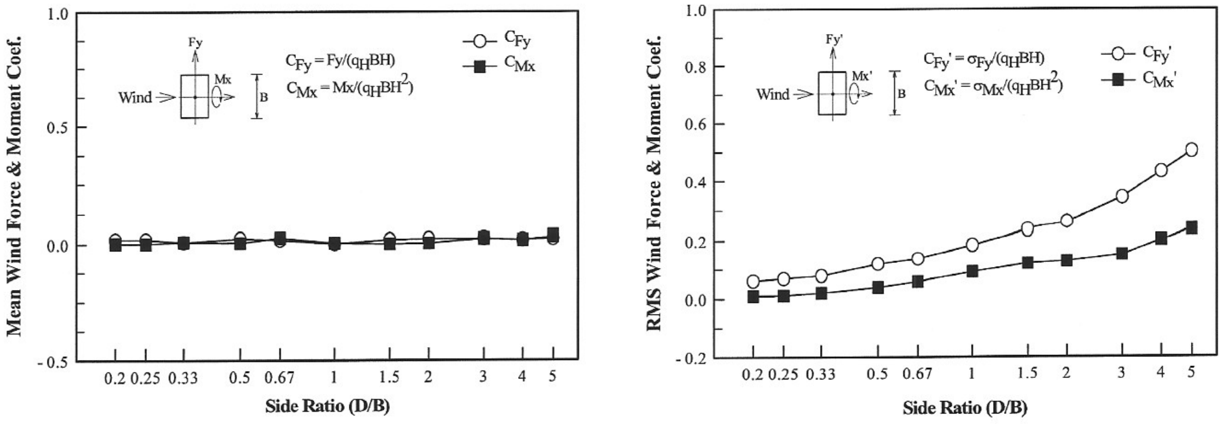
Mean wind force coefficient; $C_{Fy} = \left\{\overline{F_y}/(q_H BH)\right\}$ (1)
Mean overturning moment coefficient; $C_{My} = \left\{\overline{M_y}/(q_H BH^2)\right\}$ (2)
Fluctuating wind force coefficient; $C_{Fy}' = \left\{\sigma_{Fy}/(q_H BH)\right\}$ (3)
Fluctuating overturning moment coefficient; $C_{My}' = \left\{\sigma_{My}/(q_H BH^2)\right\}$ (4)

where $\overline{F_y}$ and $\overline{M_y}$ are the mean force and overturning moment, $\sigma_{Fy}$ and $\sigma_{My}$ are the standard deviation of the fluctuating force and overturning moment, $q_H$ is the dynamic velocity pressure at the top of building $H$ and $\rho$ is the air density.

The characteristics in frequency domain are presented force spectral density in terms of side ratio. The spectral densities were analyzed by the MEM (Maximum Entropy Method).

### 3.2 Wind force coefficient

Fig.2 shows the effect of the side ratios on the across-wind mean and fluctuating force coefficient. The mean wind force coefficients are shown almost zero value without regard to the side ratios. This means that vortex shedding having the same magnitude but the opposite direction was occurred at the rear part of the building. The fluctuating force coefficient tends to increase as the side ratio increases. Especially, this value presents a gradual increase until the side ratio reaches to $2.0$ but shows a current increase as the side ratio exceeds $3.0$.



**(a) Mean force coefficient** **(b) Fluctuating force coefficient**

**Fig.2 Across-wind force coefficient**

### 3.3 Wind force spectral density

Fig.3 shows the across-wind force spectral density as a function of the side ratio $D/B$ and the reduced frequency $nB/V_H$ where $n$ is the natural frequency of the cylinder and $V_H$ is the wind velocity at the top of building. The shapes of spectra presents narrow band. When the side ratios are less than $1.0$, there is radical one peak of the resonance by vortex shedding and the reduced frequency of spectral peak presents about $nB/V_H = 0.1$. This is caused by the vortices from the both sides shed alternatively into the wake synchronized with the across wind vibration. The height of peaks is comparatively high.

When the side ratio is large than $1.5$, there are a first gentle spectral peak in the region of the reduced frequency between $0.01$ and $0.06$. Especially, when the side ratio is larger than $3.0$, there are two spectral peaks. This is probably caused by the leading edge vortices which shed into the wake and reattach the side faces synchronized with the motion of the cylinder. The second spectral peak occurs in the region of the reduced frequency between $0.11$ and $0.16$.

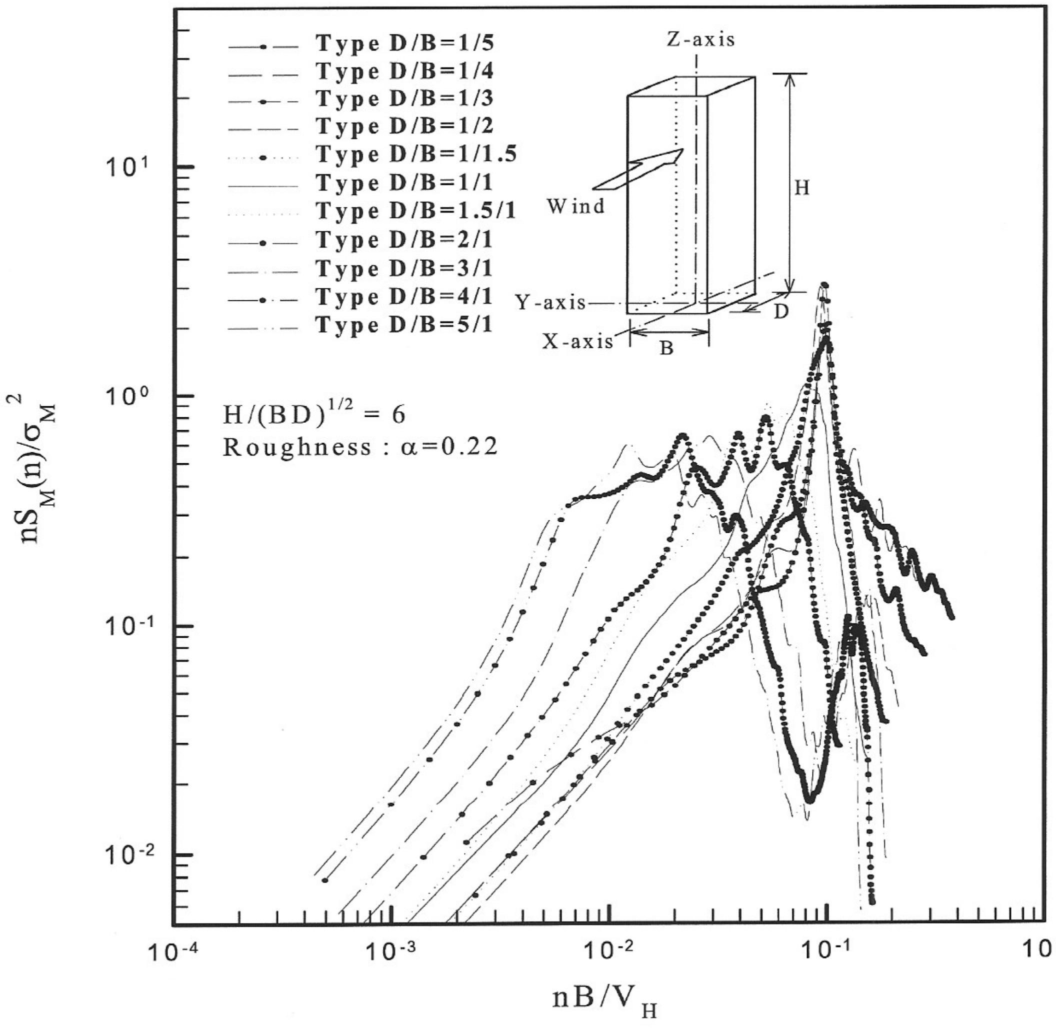


**Fig.3** Across-wind fluctuating overturning moment spectral density with various side ratios

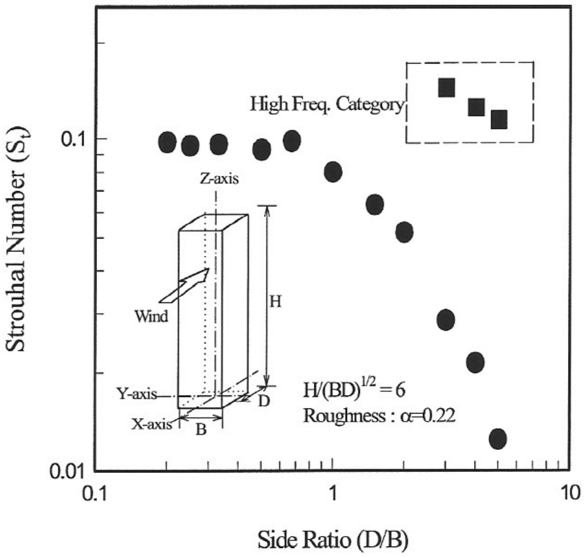


**Fig.4 Strouhal Number for across-wind**

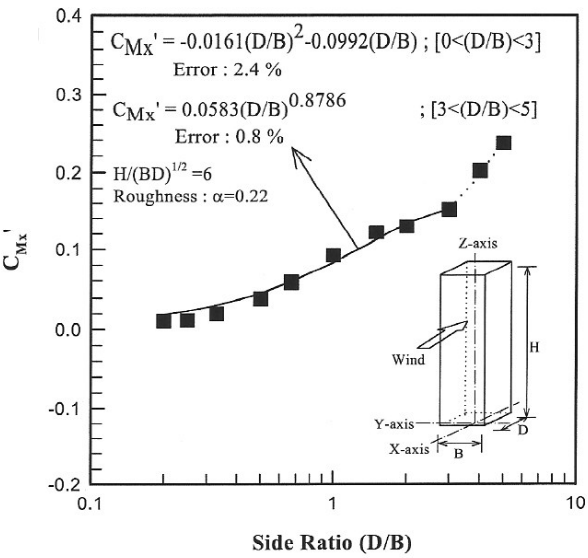


**Fig.5 Generalized across-wind fluctuating overturning moment coefficient**

Fig.4 shows the Strouhal Number $S_t$ for the across-wind plotted against the side ratios. In this paper, the Strouhal Number $S_t$, when the frequency of vortex shedding equals to the frequency of spectral peak $n_p$, is given by

$$S_t = n_p B / V_H \tag{5}$$

Strouhal Number presents between 0.01 and 0.1 in the low frequency region and this value decreases as the side ratio increases. It is interesting that, when the side ratio is less than 1.0, Strouhal Number tends slowly to decrease as the side ratio increases, but suddenly to decrease as the side ratio exceeding 1.0. On the other hand, Strouhal Number, as is shown High Freq. Category in Fig.4, corresponds with the second spectral peak in the region of the high frequency as is shown in Fig. 3.

## 4. Empirical equation

### 4.1 Fluctuating moment coefficient

Fig.5 shows the fluctuating overturning moment coefficient for the across-wind as a function of the side ratios. These tend to slowly increase below the side ratios 3.0 and suddenly increase as exceeding 3.0, so that it can be approximately estimated by dividing the side ratio into two parts as follows:

$$C_{Mx}' = -0.0161(D/B)^2 + 0.0992(D/B) \quad ; \quad [0 < (D/B) < 3.0] \tag{6.a}$$

$$C_{Mx}' = 0.0583(D/B)^{0.8786} \quad ; \quad [3.0 \le (D/B) \le 5.0] \tag{6.b}$$

where $C_{Mx}'$ is the across-wind fluctuating overturning moment coefficient. Equation (6.a) has 2.4% and equation (6.b) has 0.8% of an acceptable error range.

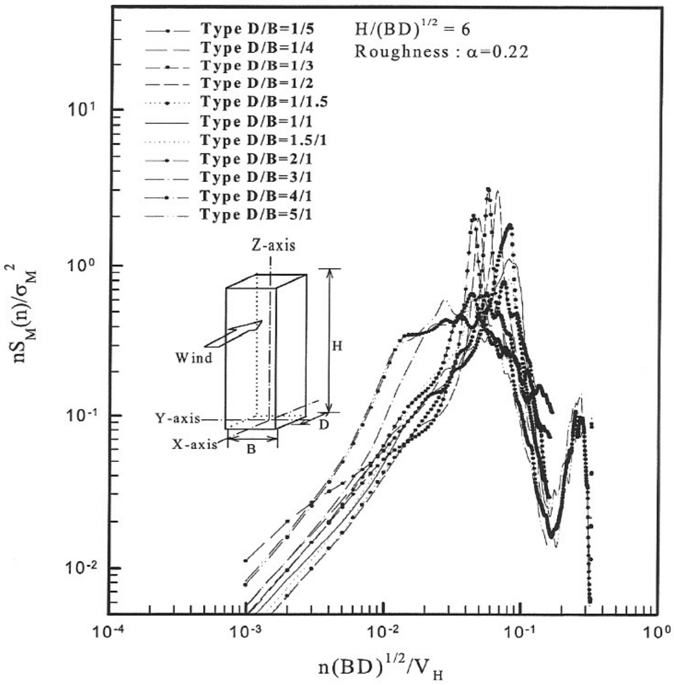


**Fig.6** **Across-wind fluctuating overturning moment spectral density with various side ratios by using reduced frequency** $n\sqrt{BD}/V_H$

### 4.2 Fluctuating moment spectral density coefficient

Fig.6 shows across-wind fluctuating moment spectral density of which the reduced frequency takes $n\sqrt{BD}/V_H$ instead of $nB/V_H$. From Fig.6, it is shown that the shape of spectra appears a clear one peak under the side ratio 2.0, but two peaks over the side ratio 3.0. Therefore, spectral density can be approximately estimated by dividing the side ratio into two parts as following:

$$F = \sum_{j=1}^{N} \frac{4x_j(1+0.6\beta_j)\beta_j}{\pi} \frac{(n/n_{sj})^2}{\{1-(n/n_{sj})^2\}^2 + 4\beta_j^2(n/n_{sj})^2} \tag{7.a}$$

$$N = \begin{cases} 1: D/B < 3 \\ 2: D/B \geq 3 \end{cases}, \qquad \begin{cases} x_1 = 0.85 \\ x_2 = 0.02 \end{cases} \tag{7.b}$$

Fig.7 shows the spectral peak frequency as a function of side ratios. From this figure, approximate equation can be estimated as follows:

$$n_{s1} = \frac{1}{-0.3474(D/B)^3 + 4.7668(D/B)^2 - 4.1473(D/B) + 11.663} \frac{V_H}{B} \tag{8.a}$$

$$n_{s2} = \frac{1}{3.6295(D/B)^{0.5}} \frac{V_H}{B} \tag{8.b}$$

Equation (8.a) has 0.5% and equation (8.b) has 0.1% of an acceptable error range.

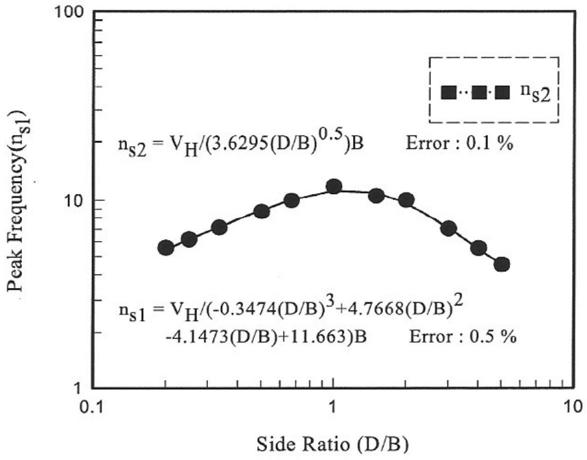


**Fig.7 Spectral peak frequency**

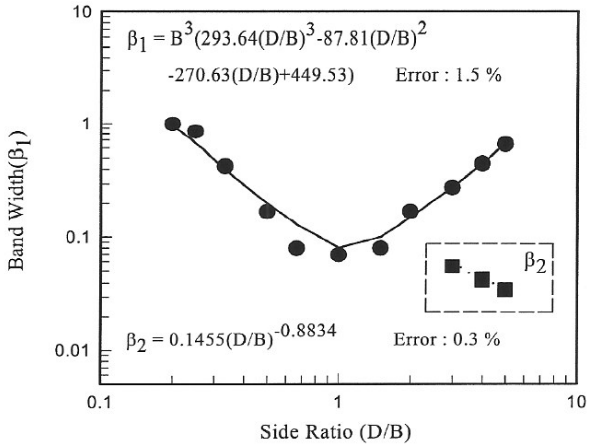


**Fig.8 Coefficient for effective spectral bandwidth**

Fig.8 shows the coefficient for effective bandwidth of spectra as a function of the side ratios.

$$\beta_1 = B^3\left\{293.64(D/B)^3 - 87.81(D/B)^2 - 270.63(D/B) + 449.53\right\} \tag{9.a}$$

$$\beta_2 = 0.1455(D/B)^{-0.8834} \tag{9.b}$$

Equation (9.a) has 1.5% and equation (9.b) has 0.3% of an acceptable error range.

Fig.9 shows the generalized across-wind fluctuating force spectral density coefficient with various side ratios which is drawn by equation (7.a) and (7.b).

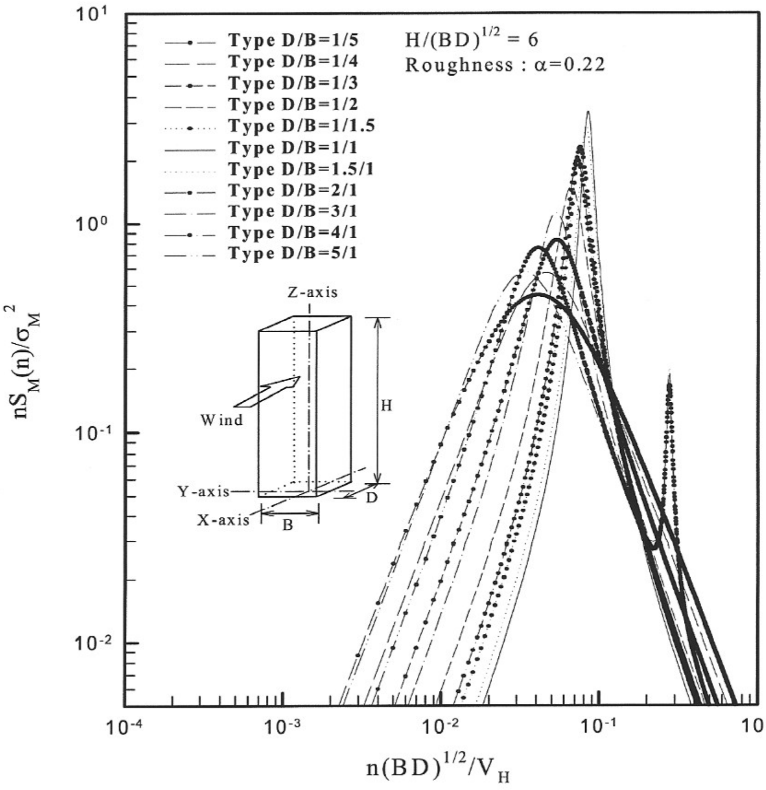


**Fig.9 Generalized across-wind force spectral density coefficient**

## 5. Conclusion

The across-wind fluctuating overturning moment coefficients tend to slowly increase with below the side ratio $3.0$ and suddenly increase with exceeding $3.0$, so approximate equations can be estimated by dividing the side ratio into two parts; below and over the side ratio $3.0$. Its value varies from $0.01$ to $0.25$.

The shapes of across-wind fluctuating moment spectral density coefficient present a clear one peak below the side ratio $2.0$ but two peaks over the side ratio $3.0$. Therefore, approximate equations can be estimated by dividing the side ratio into two parts; below and over the side ratio $3.0$. Each approximate equation is dependent on the spectral peak frequency and the effective spectral bandwidth.

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