ON THE PRELIMINARY DESIGN OF PASSIVE TUNED DAMPERS TO REDUCE WIND INDUCED ACCELERATIONS

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1.0 INTRODUCTION

The use of damping as a means of reducing wind induced motions is becoming increasingly common. While there are many systems available to increase the damping, attention here will be limited to passive tuned dampers which may have a solid or liquid moving mass that is tuned to a particular mode of the building. Building motion is "seen" by the tuned element as an excitation near resonance and hence significant motions develop and energy is dissipated by virtue of the damping elements connecting the building and the moving mass. A review of the use of damping devices with specific examples has been written by Kareem, Kijewski and Tamura[1].

While the detailed design of this type of damper may not be undertaken by many wind engineers there will commonly be instances where a preliminary design needs to be performed to assess probable costs and feasibility. It is to this end that the present paper is aimed. Two types of dampers are addressed; these are the solid tuned mass damper and the liquid "U-tube" damper. In the case of the latter device the approach taken is to define an equivalent solid mass that enables the well established theory of the tuned mass damper to be applied and produce results that are adequate for a preliminary assessment.

Section 2 of the paper reviews the theory of the tuned mass damper and presents the information necessary to size the various elements and to predict performance. Two sample designs are presented, using conventional linear shock absorbers as the damping device and one using highly non-linear wire rope coils to provide both the damping and the stiffness connection between the mass and the main structure.

Section 3 deals with "U-tube" liquid dampers while Section 4 presents a design case employing both a conventional TMD and a liquid "U-tube" damper.

The primary conclusions arising from this review are presented in Section 5.

2.0 TUNED MASS DAMPERS WITH A SOLID MOVING ELEMENT

2.1 Ideal Linear Auxiliary Mass Damper

An idealized linear tuned mass damper is shown schematically in Fig. 1a. The primary or building mass is denoted by M and the stiffness by K, the inherent building damping is represented by a linear dashpot ($F_D = C\dot{x}$) that provides damping as a fraction of critical, $\beta = C / 2\sqrt{MK}$. The primary mass is excited by the force $F(t)$ and the displacement is denoted by $x(t)$. The auxiliary mass (m) is connected to the building by a spring (k_a) and a dashpot ($\beta_a = c_a / 2\sqrt{m k_a}$. The displacement of the auxiliary mass is $x_a(t)$ and the displacement relative to the main mass is $x_r(t)$. In developing the theory an attempt will be made to replace the system of Fig 1a by a single-degree-of-freedom system having the original mass and stiffness but with the damping increased from β to $\beta + \beta_{TMD}$ (Fig. 1b). This simplification cannot be exact but it is a very convenient approach for preliminary design since wind tunnel test results will most commonly yield acceleration predictions in a given mode as a function of the damping in that mode rather than as a function of the more complex system of Fig. 1a.

The design of the tuned mass damper involves the choice of three parameters,

(i) the mass ration; $\mu = m/M$

(ii) the frequency tuning ratio; $f_r = \sqrt{k/m} \Big/ \sqrt{K/M}$

(iii) the damping ratio; $\beta_a = c_a \Big/ 2\sqrt{m\,k_a}$

The frequency response function $H(i\,f)$ (or $|H(f)|$) can be computed as a function of f/f_o where $f_o = \frac{1}{2\pi}\sqrt{K/M}$ (i.e. Warburton[2]) and typical plots are shown in Fig 2. The single degree of freedom system is replaced by a pair of peaks with a frequency separation of approximately $\pm\sqrt{\mu}/2$ about the original frequency and the magnitude of the response function falls with increasing mass ratio.

Design of the TMD involves choosing the three key parameters but for a given mass ratio there will be values of f_r and β_a which lead to optimum performance. Unfortunately, optimization depends upon the quantity to be optimized and the nature (spectral distribution) of the exciting force $F(t)$. In an elegant treatment of the problem den Hartog[3] examined the case of minimizing the rms response of the main mass to a sinusoidal force that could have any given frequency. This classical analysis was limited to the case of $\beta = 0$ and for this case the optimum values are;

$$f_{r_{opt}} = 1/(1 + \mu) \qquad 2.1$$

$$(\beta_a)_{op} = 3\mu \big/ 8(1 + \mu) \qquad 2.2$$

which yield a maximum amplification factor of $(1 + 2/\mu)^{1/2}$. Since a single degree of freedom system with damping β_e has an amplification factor of $\frac{1}{2\beta_e}$ we can write that, β_e; (the effective damping that would produce the same peak displacement) as;

$$\frac{1}{2\beta_e} = \left(1 + \frac{2}{\mu}\right)^{1/2}$$

or

$$\beta_e = \frac{1}{2\left(1 + \frac{2}{\mu}\right)^{1/2}} \qquad 2.3a$$

$$\simeq \frac{\sqrt{\mu}}{2\sqrt{2}} \quad \text{for small } \mu \qquad 2.3b$$

Warburton and Ayorinde[4] examined the influence of main system damping on the optimum values of f_r and β_a. Their numerical solutions indicate that for practical values of β (β < 2% say) the shift is of no consequence i.e. for μ = 0.10, $f_{r_{opt}}$ (β = 0) = 0.9091 and $f_{r_{opt}}$ (β = 0.02) = 0.9009.

The response to sinusoidal loading is of interest but not of great relevance in terms of the response to wind for which the excitation is, for the most part, random. A step towards random excitation was taken by Crandall & Mark[5] (1963) who examined the response of the TMD system to white noise but no closed form solutions were found. In 1970, Vickery & Davenport[6] examined the TMD response to white noise and introduced the concept of an equivalent damping that could be added directly to the original SDo F system to produce exactly the same response. This work was also done numerically but closed form solutions have since been obtained Warburton (1982). With no damping in the main system and an optimization based on the main system deflection (with white noise excitation) the optimum parameters are;

$$(f_r)_{opt} = \frac{\sqrt{1 + \mu/2}}{1+\mu} \simeq 1 - \frac{3}{4}\mu \qquad 2.4$$

$$(\beta_a)_{opt} = \left\{\frac{\mu\left(1 + 3\mu/4\right)}{4(1+\mu)\left(1 + \mu/2\right)}\right\}^{1/2}$$

$$\simeq \frac{\sqrt{\mu}}{2}\left(1 - \frac{3}{8}\mu\right) \qquad 2.5$$

The value of β_e corresponding to optimum values of β_a and f_r is obtained by equating the rms deflection of the 2 DoF system and the S DoF with a damping of β_e. This yields;

$$(\beta_e)_{opt} = \frac{\sqrt{\mu}}{4}\left(\frac{1+\mu}{1+\frac{3}{4}\mu}\right)^{1/2}$$

$$\simeq \frac{\sqrt{\mu}}{4}\left(1 + \frac{\mu}{8}\right) \qquad 2.6$$

Of further interest is the ratio, *R*, of the rms relative deflection of the auxiliary mass to that of the main mass. This ratio is;

$$R = \frac{1}{\sqrt{2\mu}} \frac{(1 + \mu)}{\sqrt{1 + \frac{3}{4}\mu}}$$

$$\simeq \frac{1}{\sqrt{2\mu}}\left(1 + \frac{5}{8}\mu\right) \qquad 2.7$$

Equations 2.4 to 2.7 define the optimum damper and its performance but for a restricted set of conditions

i.e.;

(i) $\beta = 0$, undamped main system

(ii) white noise forcing

(iii) optimization of rms deflection of main system

2.2 Deviations from Zero Main Damping

It is of interest to examine any changes or errors that are introduced if we depart from these conditions. As noted by Warburton & Ayorinde the addition of a small (< 2%) amount of damping to the main system had no significant influence on the optimum values of β_a and f_r. Gerges[7] examined the case of white noise excitation and came to similar conclusions in regard to the tuning values but did note small but significant changes in β_e. Gerges numerical study indicated that the value of β_e calculated assuming $\beta = 0$ cannot be directly added to the main system damping. His finding was that the total damping on the SDoF system was best given by,

$$\beta_{TOTAL} = \beta_e + \alpha \cdot \beta \qquad 2.8$$

where the multiplier "α" is shown in Fig. 3 as a function of β and μ. α has a value very close to ¾ for values of μ and β likely to be encountered. Gerges also noted small changes (less than 5%) in the response ratio R.

2.3 Deviations from White Noise Excitation and Optimization on Displacement

The effect of spectral shape and also optimization with respect to acceleration has not received much attention and they are best handled together. Optimization with white noise and deflection is the same as an optimization with respect to acceleration but with a force spectral density varying as f^{-4} rather than constant. In terms of wind engineering a force spectral density varying as f^{-4} is far closer to the truth that a flat spectrum (at least in the vicinity of , f_o, a typical building frequency).

A study by Vickery (unpublished report, 2001) examined the influence of spectral form on the optimization parameters and the "added damping" β_e. The mass ratio was $\mu = 0.16$ and hence the predicted values would be;

From 2.4 $\quad (f_r)_{opt} = 0.896$

2.5 $(\beta_a)_{opt}$ = 0.189

2.6 $(\beta_e)_{opt}$ = 0.102

The analysis was based upon acceleration and hence a spectrum varying as f^{-4} should yield close to the above. The spectra examined were of the form $200 \Big/ \left(1 + 200\left(f/f_o\right)^N\right)$ with N = 2 and 3. The computed values of the above three parameters were,

N = 2 $(f_r)_{opt}$ = 0.928

$(\beta_a)_{opt}$ = 0.205

$(\beta_e)_{opt}$ = 0.105

N = 3 $(f_r)_{opt}$ = 0.910

$(\beta_a)_{opt}$ = 0.195

$(\beta_e)_{opt}$ = 0.111

Both the above cases have 1% main system damping but 1.5% produced similar results. This study was far from comprehensive but it would appear that the results presented in Equations 2.4 to 2.7 provide a sound basis for the design of dampers that are optimized to minimize wind induced building accelerations.

2.4 Application to Real Building

The system shown in Fig. 2.1 is an idealized system with only two degrees of freedom. In the case of a real building with a TMD attached there are many modes and the simplifications involved in reducing it to a two-degree-of-freedom system deserve attention. For a building with two distinct sway modes and a torsional mode it is possible to simply reduce the problem to a set of three 2 DoF systems. In the case of a translational or sway mode the main mass M of the system depicted in Fig. 2.1 is

$$M = \frac{\int_o^H m(z)\phi^2(z)dz}{\phi^2(z_A)} \qquad 2.9$$

$\phi(z)$ = mode shape as a function of height z

$m(z)$ = mass per unit height

z_A = height of damper

K = $M(2\pi f_o)^2$

This "single mode approach" has been discussed by Warburton & Ayorinde who examined "single mode" and "multi-mode" approaches for simply supported beams and plates. The primary conclusions are that the "single mode" approach is adequate for the structures studied even at mass ratios as high as 1.0. Typical errors in $(f_r)_{opt}$ at μ = 0.20 (a likely upper limit for buildings) were about 10% and errors in $(\beta_a)_{opt}$ somewhat less. The errors were markedly less at lower mass ratios but the sensitivity of performance on deviations from optimum increases with decreasing mass ratio.

2.4 Deviations from Optimum Tuning

For many reasons a damper as constructed will not match the optimum tuning conditions. This may be due to less than perfect definition of the damper properties or, more likely, less than perfect definition of the building frequency. For this reason it is worthwhile to examine the off-optimum performance of a TMD.

For the idealized system of Fig. 2.1 and optimized for minimum rms deflection with white noise or minimum rms acceleration with a f^{-4} spectrum the off-tune performance is given by the following equations;

$$\beta_e = \mu f_r \beta_a R^2 \qquad 2.10$$

$$R^{-2} = f_r^4 (1+\mu)^2 + f_r^2 \left(4\beta_a^2 (1+\mu) - 2 - \mu\right) + 1 \qquad 2.11$$

$$R = \frac{\textit{relative rms disp of auxiliary mass}}{\textit{rms disp of main mass}}$$

Equation 2.10 is plotted in Fig 4 for a near optimum value of β_a of $\sqrt{\mu}/2$. This figure clearly demonstrates the reduction in damper performance as the frequency ratio $\left(f_r = f_a/f_o\right)$ is moved from its optimum value. The frequency range is dependent on the mass ratio and for a value of β_e no less than $\alpha (\beta_e)_{op}$ the range is given quite closely by

$$\alpha = 0.80, \quad f_r = (f_r)_{op} \left\{1 \pm 0.35\sqrt{\mu}\right\} \qquad 2.12$$

$$\alpha = 0.90, \quad f_r = (f_r)_{op} \left\{1 \pm 0.25\sqrt{\mu}\right\}$$

$$\alpha = 0.95, \quad f_r = (f_r)_{op} \left\{1 \pm 0.17\sqrt{\mu}\right\}$$

Values of $\beta_e/(\beta_e)_{op}$ for a near optimum tuning ratio of $f_r = 1/(1+\mu)$ are shown in Fig. 5.

$$\frac{\beta_e}{(\beta_e)_{op}} = \frac{2\left(\beta_a/(\beta_a)_{op}\right)}{\left(\beta_a/(\beta_a)_{op}\right)^2 + 1} \qquad 2.13$$

and

$$\frac{R}{R_{opt}} = \sqrt{2}\left((1+\mu)\Big/\left(1+\left(\beta_a\Big/\beta_{a_{op}}\right)^2\right)\right)^{1/2} \qquad 2.14$$

2.5 Peak Accelerations

The rms acceleration at a given position due to motion in a particular mode is given very closely by;

$$\sigma_a = \frac{(2\pi f_o)^2}{K}\left(\frac{\pi f_o\, S(f_o)}{4\beta}\right)^{1/2} \cdot \phi(z_a) \qquad 2.15$$

$S(f_o)$ = spectral density of modal force

K = modal stiffness

$\phi(z_a)$ = modal displacement at the given position

β = damping as a fraction of critical and including the damping β_e from the TMD and also the inherent structural damping and any aerodynamic damping

Equation 2.15 is not dependent upon the distribution of the modal force but the peak acceleration is and the peak factor may be a function of damping if the distribution of the force is not Gaussian as is commonly the case at high turbulence levels. This is not a problem if the relationship between acceleration and damping comes from aeroelastic testing but with the use of pressure data or base balance data an analysis in the time domain is advised to check the dependency of the peak factor on damping.

An example of the dependence of the peak factor on damping is presented in Figs 6 and 7. Fig 6 shows the dependence of the rms acceleration of a building on damping. The results shown are from time history simulations of the response of an actual building to wind load. The load histories were derived from a roof mounted anemometer after the application of a filter to account for the estimated aerodynamic admittance of the building. Results are presented for three 20 minute segments and the simulated accelerations for a range of damping values are shown along with actual field measurements at the estimated full scale damping of 1.25% of critical. As expected, the rms values vary inversely with the square-root of the damping but, as shown in Fig 6, the peak factor rises from 5 to 8 over the same (1% to 10%) damping range. These change in damping reduced the rms accelerations by close to 70% but the peak accelerations were reduced by only 50% for a tenfold increase in damping. This is a very significant loss of efficiency and would require an increase in the TMD mass of $(8/5)^2$ to obtain the anticipated reductions with a constant peak factor.

3.0 TUNED "U-TUBE" LIQUID DAMPER

The liquid tuned damper shown in Fig 8 is of the "U-tube" type and the equations of motion can be expressed in a form very similar to those for a TMD. The following analysis uses a simple one-dimensional flow analysis and a co-ordinate adjustment to develop the equations of motion which are identical to a TMD with non-linear (V^2) damping. The analysis is not exact since various parameters (such as the head losses in the system) must be estimated but the results are well suited to a preliminary design. Final design should be based on model tests.

For the LTMD shown in Fig 8, the underlying equation is;

$$\frac{\partial H}{\partial s} = -\frac{1}{g}\frac{\partial V(s,t)}{\partial t} - S_f;$$

S_f = friction slope

Integrating the equation from position 1 to position 2;

$$\int_1^2 dH = Z_2 - Z_1$$

$$= -\frac{1}{g}\int_1^2 \frac{\partial}{\partial t}\left(V(s,t) + \dot{x}_1 \frac{dx}{ds}\right) ds$$

$$- \int_1^2 S_f \cdot ds \int$$

Putting

$$V(s) = V_o \cdot \frac{A_o}{A(s)}$$

$$Z_2 - Z_1 = -\frac{L_e}{g}\frac{dV_o}{dt} - \frac{L_o}{g}\ddot{x}_1 - C_L \frac{V_o^2}{2_g}$$

where;

$$L_e = \int_1^2 \frac{A_o}{A(s)} \cdot ds$$

$$L_o = \int_1^2 \frac{A_o}{A(s)} \cdot dx$$

$C_L \frac{V_o^2}{2_g}$ = total friction loss between 1 and 2

C_L = head of loss coefficient

V_o = average velocity through horizontal duct

From continuity of incompressible flow;

$$A_o V_o = A_R \frac{dz_2}{dt}$$

and from conservation of mass within the LTMD;

$$Z_1 + Z_2 = 0$$

Defining s_o as the relative displacement of fluid in the horizontal duct, the equation of motion becomes;

$$2\frac{A_o}{A_R} s_o = -\frac{L_e}{g}\ddot{s}_o - \frac{L_o}{g}\ddot{x}_1 - C_L \frac{V_o |V_o|}{2_g}$$

If we define a new variable $x_r = s_o \dfrac{L_e}{L_o}$ or $s_o = \dfrac{L_o}{L_e} x_r$, the equation may be rewritten as;

$$\frac{L_o}{g}\ddot{x}_r + C_L \left(\frac{L_o}{L_e}\right)^2 \dot{x}_r |\dot{x}_r| \Big/ 2g = -\frac{L_o}{g}\ddot{x}_1 + 2\frac{A_o L_o}{A_R L_e} x_r$$

Multiplying by $\rho A_o g$ we have;

$$\rho A_o L_o \ddot{x}_r + \frac{A_o}{2}\rho C_L \left(\frac{L_o}{L_e}\right)^2 \dot{x}_r |\dot{x}_r| + \frac{2\rho A_o g L_o}{A_R L_e} x_r = -\rho A_o L_o \ddot{x}_1$$

The equation of motion for a conventional TMD (see Fig. 2) is;

$$m\,\ddot{x}_r + c\,\ddot{x}_r \,|\, \ddot{x}_r \,| + k\,x_r = -\,m\,\ddot{x}_1$$

Thus, the established theory for conventional TMD's is applicable to the LTMD with the following substitutions

$$m = \rho A_o L_o$$

$$x_r = x_o \frac{L_e}{L_o}$$

$$k = 2\rho \frac{A_o^2 \, g \, L_o}{A_R \, L_e}$$

$$c = \rho \frac{Ao}{2} C_L \left(\frac{L_o}{L_e}\right)^2$$

The natural frequency of the LTMD is given by

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{2 g A_o}{L_e A_R}}$$

The non-linear TMD equation can be approximated by a linear form

$$m\ddot{x}_r + c_e\,\ddot{x}_r + k\,x_r \quad = \quad -\,m\,\ddot{x}_1$$

where

$$c_e \;=\; 2\sqrt{mk}\;\beta_2$$

and

$$\beta_2 \;=\; \sqrt{\frac{2}{\pi}}\cdot\frac{c\,\sigma_r}{m}\cdot f_r$$

σ_r = rms value of x_r

4.0 DESIGN CALCULATIONS

4.1 Preliminary Considerations

The building for which a damper system will be designed is shown in Fig. 9. It is presumed that for the anticipated damping of 1% of critical the predicted peak accelerations are

$\hat{a}_x$ = 15 $(V/20)^3$ milli-g

$\hat{a}_y$ = 10 $(V/20)^3$ milli-g

$\hat{a}_o$ = 12 $(V/30)^3$ milli-g at 25 m from the centre.

and the relationship between the wind speed V and the return period (R) is;

V = 20 (1 + 0.14 ln R)

The aim of the design will be to size a damping system that will yield a peak corner acceleration of no more than 10 milli-g for R = 1 year and will perform acceptably for speeds up to R = 20 years and be safe for R = 350 years. The three speeds to be considered are

R = 1 $\qquad V$ = 20 m/s

R = 20 $\qquad V$ = 1.42 x 20 m/s

R = 350 $\qquad V$ = 1.82 x 10 m/s

For the purposes of this example it will be assumed that the corner acceleration is, very conservatively, equal to the square root of the sum of the squares of the three components, i.e.

$$\hat{a} = (15^2 + 10^2 + 12^2)^{1/2} \quad (V/20)^3$$

$$= 21.7\,(V/20)^3$$

In order to reduce this to 10 milli-g the total damping should be increased to 2.17^2 x 1 = 4.71%. The damping system should provide β_e where

$$0.047 = \beta_e + 0.75 \times 0.01$$

or $$\beta_e = 0.0396$$

It is proposed use a conventional solid TMD to control X and Y accelerations and a pair of "U-tube" dampers to control torsional motions. In order to allow for imperfect tuning it is necessary to assume an efficiency that will depend upon the type of damping system employed. Solid TMD's can be more closely controlled than the non-linear liquid dampers and it suggested that an efficiency of 90% would be appropriate for the solid TMD and 75% for the U-tube.

In the case of the solid damper the target β_e is 0.0396/0.9 (=0.044) which requires a mass ratio of $(4 \times 0.044)^2$ or μ = 0.031. For the liquid damper the target β_e is 0.0396/0.75 (=0.053) and the required mass ratio is 0.0449.

For a roof mounted damper the modal mass for X and Y motions is 130 x 30 x 40 x 200/3.4 or 9.18×10^6 kg and hence the required TMD mass is 285 tonne. The modal "mass" for the torsional mode is 2.066×10^9 kgm^2 and hence the TMD "mass" is 92.7×10^6 kgm^2. It will be assumed that a pair of U-tube dampers will be positioned 18m from the building centre and hence the mass per unit is given by,

$$2m\ 18^2 = 92.7 \times 10^6$$

$$m = 142 \text{ tonne}$$

4.2 Design of Solid TMD

Fig 10 depicts a pendulum TMD with a mass of 250 tonne that is suspended by four wire ropes. Three linear dashpots provide the damping (2 in Y, 1 in X direction). A "tuning frame" shortens the effective pendulum length in one direction. The optimum tuning ratio for the solid damper (Equation 2.4) is 1 – ¾ x 0.031 or 0.977 and hence the TMD frequencies are 0.176 Hz in the X and 0.195 Hz in the Y direction. The corresponding pendulum lengths are 8.03 m and 6.50 m respectively with a 1.53 m deep tuning frame. The results shown in Fig. 4 indicate the following approximate relationships between efficiency and off optimum tuning;

Efficiency 80%+ : $$f_r = (f_r)_{op} \left(1 \pm 0.35 \sqrt{\mu}\right)$$

Efficiency 90%+ : $$f_r = (f_r)_{op} \left(1 \pm 0.25 \sqrt{\mu}\right)$$

Efficiency 95%+ : $$f_r = (f_r)_{op} \left(1 \pm 0.17 \sqrt{\mu}\right)$$

For an efficiency of 95% and μ = 0.031 the acceptable frequency range is ± 3% about optimum. To ahcieve this accuracy the building frequency would be measured rather than computed.

The optimum damping for the TMD (from Equation 2.5) is 0.087 as a fraction of critical and, assuming two dashpots per direction, the linear dashpot constant is,

"X" $$C = 2 \times 285{,}000 \times 0.087 \times 2\pi \times 0.176/2$$

$$= 27.4 \text{ kN/(m/s)}$$

"Y" $$C = 30.4 \text{ kN/(m/s)}$$

The efficiency if not strongly dependent on the value of "C" and a common value of 30 kN/(m/s) for all four units would not have a measurable influence. For a return period of 20 years the load capacity, stroke and power dissipation capacity should be acceptable and have a reasonable "safety factor". The peak building accelerations for a 20 year return period and 1% damping are;

for X:	42.9 milli-g
for Y:	28.6 milli-g

Fro the TMD at optimum the total damping will be 0.051 and hence the building accelerations will be reduced to 19.0 milli-g and 12.7 milli-g respectively. The ratio of the TMD acceleration to the building acceleration is (from Equation 2.7) 4.09 and hence the TMD accelerations are 78 milli-g and 52 milli-g for X and Y. The peak values of acceleration velocity and displacement of the TMD are;

"X"	peak acceleration	0.765 m/s^2
	peak velocity	0.692 m/s
	peak displacement	0.627 m
"Y"	peak acceleration	0.510 m/s^2
	peak velocity	0.416 m/s
	peak displacement	0.340 m

The peak loads in the damping units would be 20.8 KN and 12.5 kN for X and Y respectively. The average rate of energy dissipation (assuming that the peak values are 3.7 x rms) is 1050 watts for X and 380 watts for Y. The peak displacement for the X direction might pose a problem since it requires a total stroke of about 1.5 m. This could be reduced to about 1.0 m by mounting the X dashpots at 45° to the horizontal and increasing the damper constant from 30 kN/(m/s) to 60 kN/(m/s). A second approach would be to switch to a V^2 damper tuned for optimum performance at about R = 2 years. This would increase the building accelerations at R = 20 years but reduce the stroke requirement. Inclined V^2 dampers would reduce the stroke requirements even further and the choice of a final arrangement will depend on the costing of the various alternatives.

4.3 Design of Liquid "U-tube" Damper

The design mass ratio is μ = 0.0449 using two 143 tonne units spaced 36m apart on the roof. The target frequency for optimum performance is 0.250/1.034 or 0.242 Hz and the optimum damping at the chosen point of optimization is 0.104. Fig. 11 shows an idealized U-tube for which L_e is approximated as;

$$L_e = L_o + (H+2R)(H/W) \text{ m}$$

and the frequency by;

$$f_o = \frac{1}{2\pi}\left(\frac{2g}{L_e}\frac{H}{W}\right)^{1/2}$$

$$= 0.242 \text{ Hz}$$

The mass of the damper is;

$$M = \rho H L_o B = 143{,}000 \text{ kg}$$

$$H L_o B = 143 \text{ m}^3$$

The design is commenced assuming L_o = 15.0 m, H = 2.5 m and hence B = 3.8 m. It is further assumed that R = 0.50 m and hence;

$$L_e = 15.0 + 3.5\, H/W$$

and

$$(2\pi f_o)^2 = 2gH/(W L_e)$$

from which;

$$\frac{H}{W} = 3.0, \quad W = 0.83 \text{ m}$$

$$L_e = 25.5 \text{ m}$$

$$L_e/L_o = 1.70$$

It is chosen to optimize for R = 1 year and check the performance at R = 20 years. The specified peak acceleration 18 m from the building centre is 12 x 18/25 or 8.6 milli-g at 0.25 Hz and 1% damping. This corresponds to a peak displacement of 0.034 m. At optimum damping of 0.06 this displacement is reduced to 14 mm. The ratio of x_r to x_1 at optimum is approximately $1/\sqrt{2\mu}$ and hence the peak value of x_r = 47 mm. The maximum displacement in the horizontal duct $\frac{L_o}{L_e} x_r$ or 27.5 mm and hence the maximum rise in the risers is 82 mm. The optimum damping of the U-tube is 0.104. From Section 3.0 we have,

$$\beta = 0.104 = \sqrt{\frac{2}{\pi}} \cdot \frac{C\,\sigma_r}{m} \cdot \frac{f_2}{f_1}$$

$$\sigma_r = 0.047/3.7$$

$$m = 143{,}000$$

$$f_2/f_1 = 0.97$$

and hence

$$C = 1.5 \times 10^6$$

$$C_L = 900$$

The maximum displacement in the duct of 27.5 mm corresponds to a velocity 0.043 m/s and a velocity pressure 0.92 Pa and a pressure drop across a screen of 900 x 0.92 or 828 Pa and a total force of 7.9 kN. Alternately a loss device could be placed across each riser. The velocity at this point would be three times higher and the screen loss coefficient is reduced to 50. The total force acting on each screen or perforated plate is 1.3 kN.

Calculations for $R \simeq 100$ years indicated a maximum rise of 205 mm and a maximum force on the loss screens fitted into the risers of 2.2 kN per metre.

5.0 CONCLUSIONS

The aim of the paper was to present in a compact form the relationship required for the preliminary design of tuned mass dampers in general and also the "U-tube" liquid tuned damper. Attention is drawn to differences between the ideal 2 DoF tuned damper and a tuned damper fitted to a continuous multi-degree-of-freedom system. Attention is also directed to the influence that a TMD or other added damping can have on the peak as opposed to the rms acceleration levels. In cases of high turbulence the reduction in peak acceleration may be significantly less than the reduction in rms.

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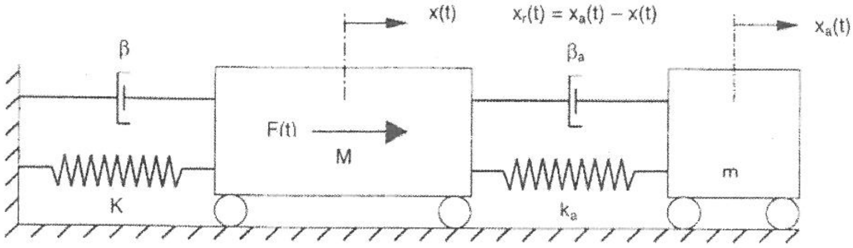
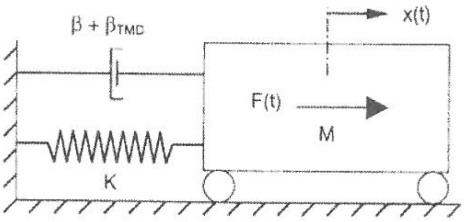
FIGURES



(a) Linear two-degree of freedom system



(b) Equivalent linear single degree of freedom system

Fig. 1: Idealized TMD system and Equivalent SDoF System

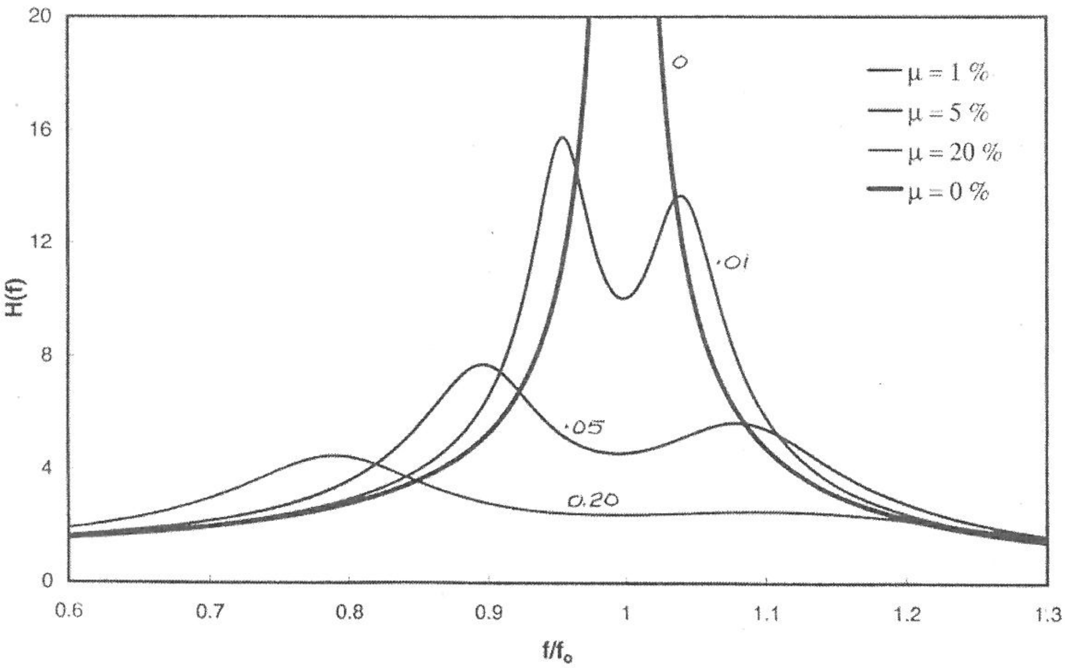


Fig. 2: Typical Frequency Response Curves of a System with a TMD with Mass Ratios of 0, 0.01, 0.05 and 0.20

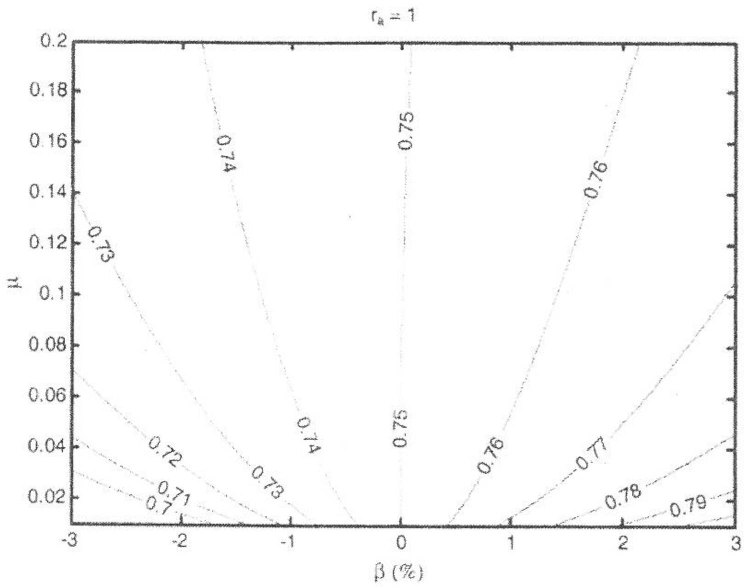


Fig. 3: Values of α in Equation 2.8 as a Function of Mass Ratio (μ) and Main System Damping (β)

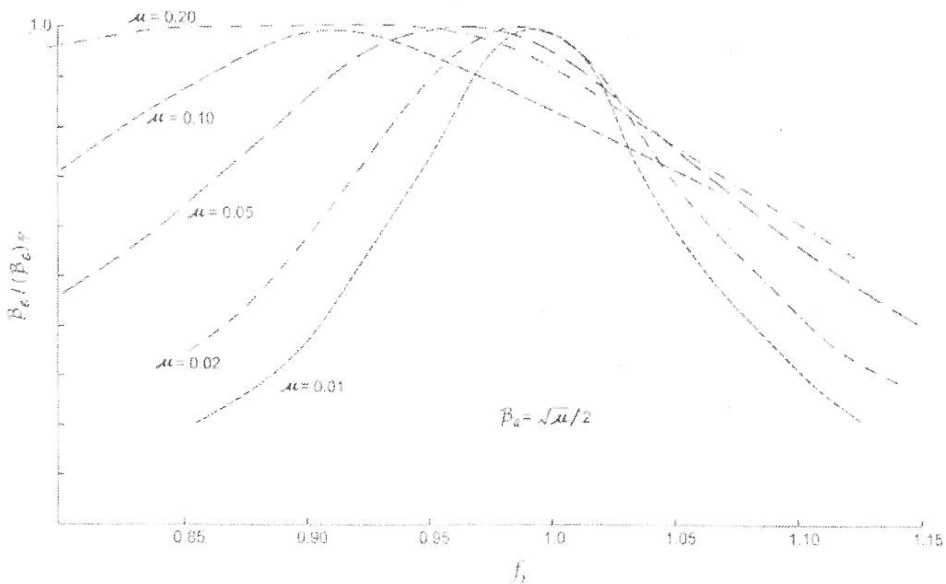


Fig. 4: Values of $\beta_e / (\beta_e)_{op}$ as a Function of the Mass Ratio (μ) and the Tuning Ratio (f_r) for near Optimum Damping $\left(\beta_a = \sqrt{\mu}/2\right)$

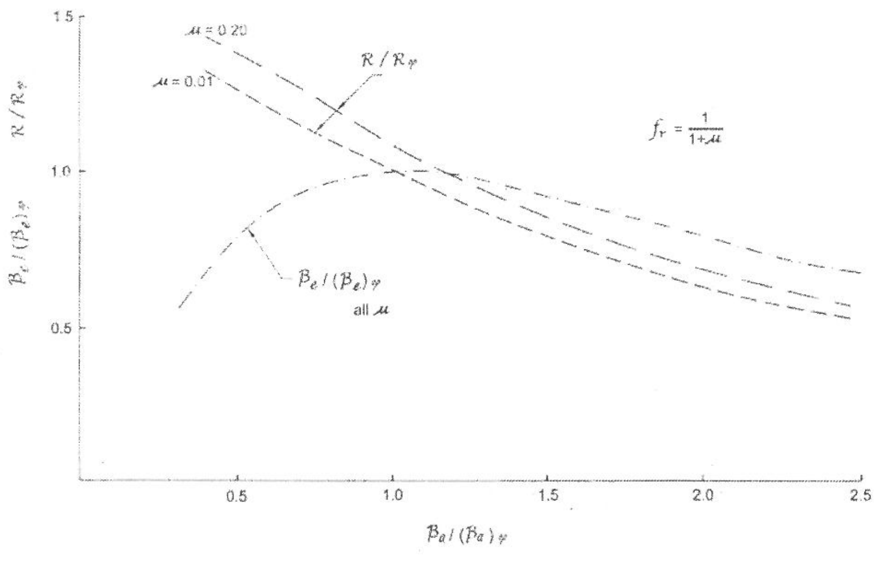


Fig. 5: Values of $\beta_e/(\beta_e)_{op}$ and R/R_{op} as a Function of the Mass Ratio (μ) and the Damping Ratio $\beta_a/(\beta_a)_{op}$ for near Optimum Tuning $\left(f_r = (1+\mu)^{-1}\right)$

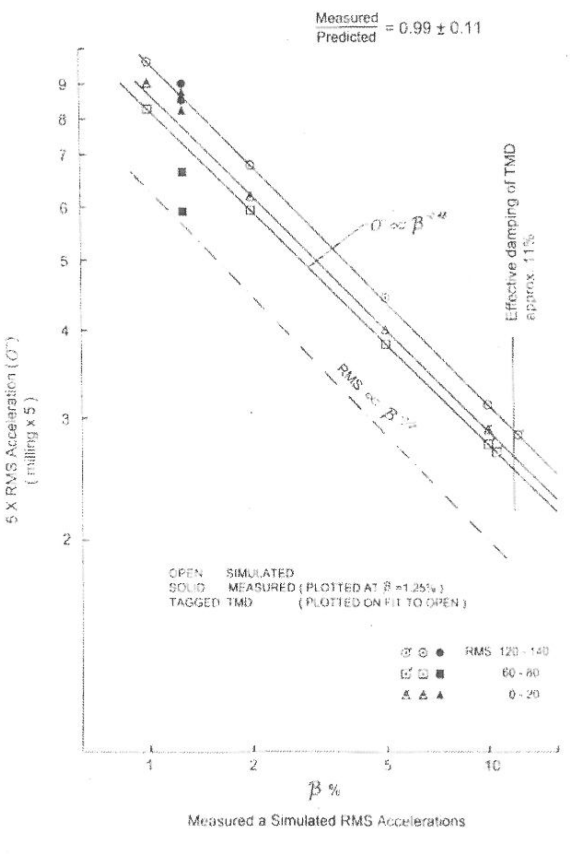


Fig. 6: Simulated and Measured Accelerations as a Function of Damping (Full Scale)

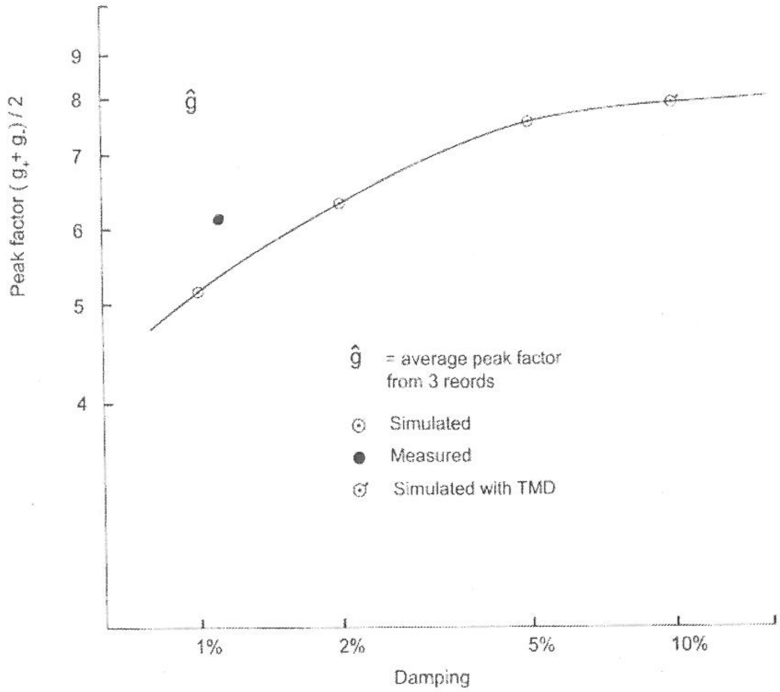


Fig. 7: Simulated and Measured Peak Factors as a Function of Damping (Full Scale)

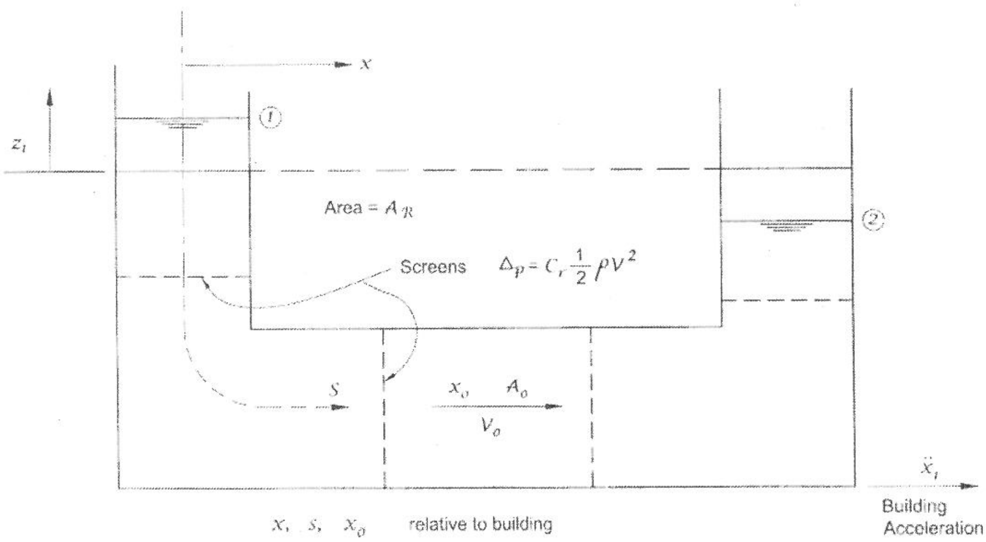


Fig. 8: Definition Sketch for a "U-tube" Liquid Tuned Damper

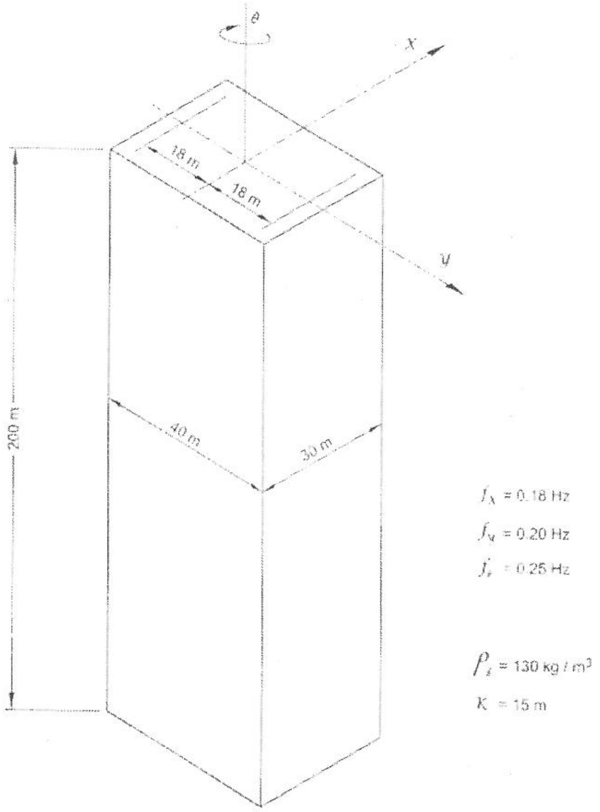


Fig. 9: Building Properties for Sample TMD Design

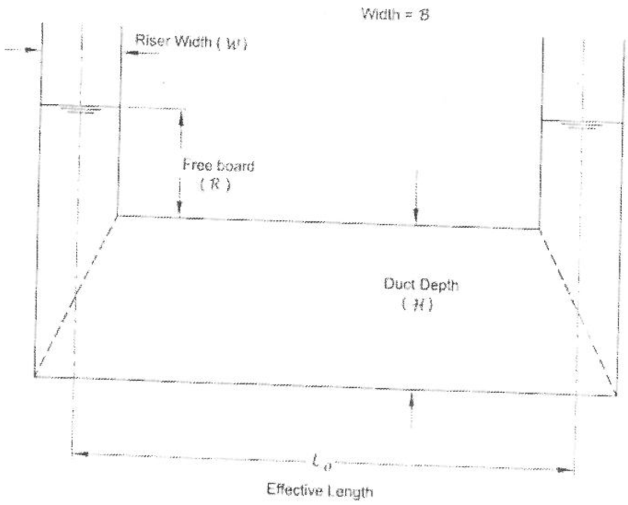
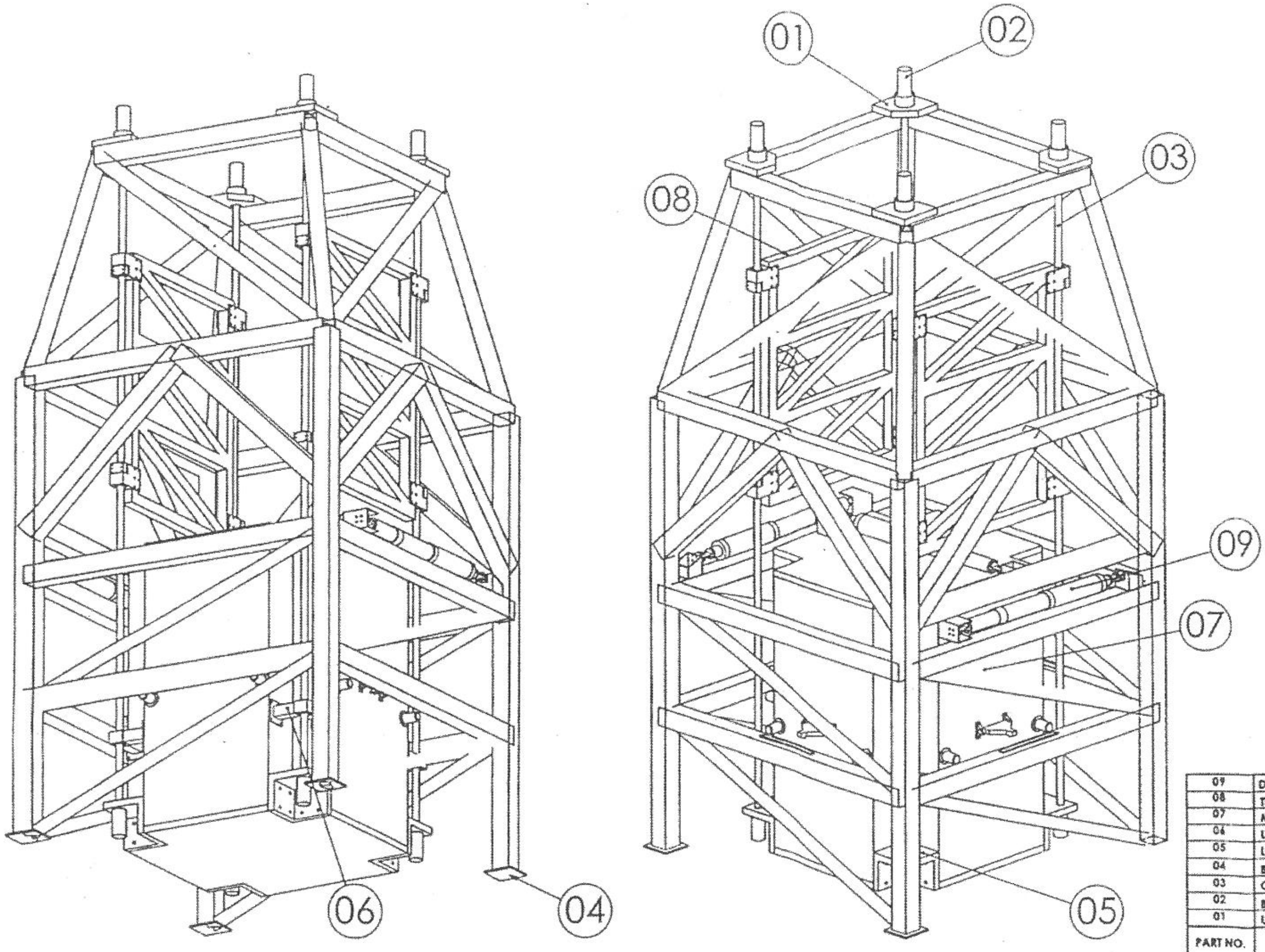


Fig. 11: Definition Sketch for a "U-tube" Liquid Tuned Damper



NOTE: DRAWING MEANT TO SHOW GENERAL ARRANGEMENT OF DAMPER FRAME AND DAMPER COMPONENTS ONLY. FOR DETAILS, SEE DRAWINGS FOR INDIVIDUAL COMPONENTS.

PART NO.	DESCRIPTION	MATERIAL	QTY
09	DAMPING DEVICE (SEE MANUF. DWG FOR DETAILS)		
08	TUNING ASSEMBLY (SEE TUNING ASSEMBLY AND PRT DWGS FOR DETAIL		
07	MASS BOX (SEE MASS BOX ASSEMBLY AND PRT DWGS FOR DETAILS)		
06	LOWER ROPE GUIDE	STEEL	4
05	LOWER ANCHOR BRACKET	— STEEL	4
04	BASE PLATE	STEEL	4
03	CABLE (WIRE ROPE)	STEEL	4
02	BUTTON STOP	STEEL	4
01	UPPER CABLE SUPPORT	STEEL	4

Boundary Layer Wind Tunnel

Kowloon Damper Unit - 3 Damper Unit

Drawn by	Checked by	Date	Scale	Dwg. Name		Pcs
TK	Dr.B.Vickery	01.04.12	1:55	AssemDmpr03	Dwg # 08	

Fig. 10: A 250 Tonne Damper Designed to Limit Accelerations in Two Directions