Reduction of Wind-Induced Vibration of Tall Buildings Using Suspended Mass Pendulums

Jong-Dar YAU, Ting-Chi HSU, and Alex TUAN

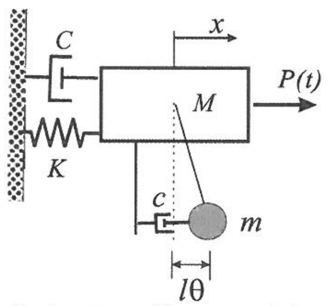
College of Technology, Tamkang University, 10620 Taipei, Taiwan

Abstract

This paper is aimed at the investigation of using suspended mass pendulums to reduce wind-induced vibration of tall buildings. The tall building is modeled as a shear building with the same floor mass and lateral stiffness for each story, and the suspended mass pendulum as a single-degree-of-freedom (SDOF) absorbing system with a lumped mass hanged by a mass-less wire. Using the minimum-maximum amplitude criterion, one can determine the optimum parameters of the suspended mass pendulum for suppressing the steady-state harmonic response of the tall building under external excitations. By adopting the Davenport wind spectrum to describe wind excitations, the vibration analysis of a tall building with 52 stories is employed to demonstrate the feasibility of the mass pendulum on mitigating the wind-induced vibration of the building. According to numerical investigations, mass pendulums are useful devices to suppress the wind-induced vibration of a tall building.

Introduction

Due to the rapid expansion of economics in some of Asia countries, many high-rise buildings have been constructed. In addition, with the advances of construction techniques and the development of high-strength and lightweight material, the dynamic behaviours of high-rise buildings would become more flexible and sensitive under the action of wind forces. Thus to meet the design requirements of serviceability and to improve the living comfort from building occupants, the wind-excited vibration of the tall buildings could be mitigated by either passive or active control devices [1-3]. In this paper, the investigation of using suspended mass pendulums (SMP) to reduce wind-induced vibration of tall buildings will be presented. By modelling a high-rise building as a plane shear building and adopting the Davenport wind spectrum to describe the wind excitations acting on a shear building, one can determine the dynamic response of the tall building with 52 stories installed with a suspended mass pendulum due to wind-induced forces. According to numerical investigations, suspended mass pendulums are useful devices to reduce the wind-induced vibration of a tall building.



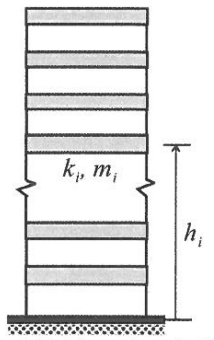


Fig. 1 Generalized system with a suspended mass pendulum. Fig. 2 Uniform shear building model.

Optimum parameters of a suspended mass pendulum

Fig. 1 shows that a suspended mass pendulum is attached to a SDOF structure excited by a harmonic force: $P(t) = P_0 e^{i\omega t}$. Based on the consideration of small rotation of the mass pendulum, the equations of motion for such a coupled vibrating system are

$$(m+M)\ddot{x}+ml\ddot{\theta}+C\dot{x}+Kx=P(t),$$
$$ml(\ddot{x}+l\ddot{\theta})+cl^2\dot{\theta}+mgl\theta=0 \qquad (1a,b)$$

where an over-dot denotes differentiation with respect to time t, x the displacement of the main mass, θ the rotational angle of the pendulum, and M, C, K represent the mass, damping coefficient, and stiffness of the main system, respectively. As for the parameters of m and c, they are the mass, damping coefficient of the mass pendulum. Thus, the steady-state response (x,θ) of the main mass and suspended mass can be expressed as

$$x=Xe^{i\omega t},\theta=\Theta e^{i\omega t} \qquad (2a, b)$$

Then substitution Eqs. (2a) and(b) into Eqs. (1a) and (1b) yields

$$\begin{bmatrix} K-(m+M)\omega^2+iC\omega & -m\omega^2 \\ -m\omega^2 & mg+ic\omega-m\omega^2 \end{bmatrix}\begin{Bmatrix} X \\ l\Theta \end{Bmatrix}=\begin{Bmatrix} P_0 \\ 0 \end{Bmatrix} \qquad (3)$$

Let ω_t denote the frequency of vibration of the mass pendulum, i.e., $\omega_t=\sqrt{g/l}$. Also, define the following parameters: f = the tuning frequency ratio ω_t/Ω, j = the exciting frequency ratio ω/Ω, μ = the tuning mass ratio m/M, and $\xi=C/(2\Omega M)$ = the damping ratio of the main system, $\zeta=c/(2\omega_t m)$ = the damping ratio of the mass pendulum. By solving Eq. (3), the amplitude R of the steady-state dynamic response X of the main mass M can be obtained as

$$R(j,f,\zeta,\mu,\xi)=\frac{P_0}{K}\times\sqrt{\frac{F_1^2+F_1^2}{F_3^2+F_4^2}}$$
$$F_1=1-(j/f)^2, F_2=2\zeta j/f, F_3=\frac{j^4-j^2}{f^2}-j^2(1+\mu)-4\frac{\zeta\xi j^2}{f}+1, \qquad (4a\text{-}e)$$
$$F_4=2j\left[\frac{j^2[\xi/f+\zeta(1+\mu)]-\zeta}{f}-\xi\right]$$

To obtain the optimum parameters of the suspended mass pendulum, using the criterion of *minimum-maximum dynamic amplification factor* [4], by which the absorbing system is optimised such that the maximum response amplitude of the main mass is minimized.

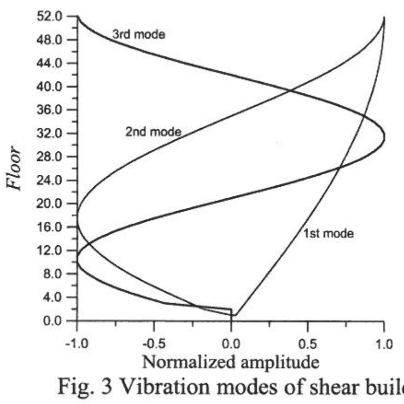


Fig. 3 Vibration modes of shear building.

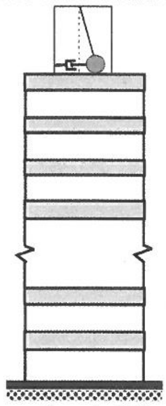


Fig. 4 Uniform shear building with a SMP.

Equations of motion

As shown in Fig. 2, a tall building regarded as a multi-degrees-of-freedom shear building system is subjected to the action of wind loads, in which a suspended mass pendulum is installed at the rigid penthouse located on the top floor of this building. The equations of motion of the vibrating system can be written as

$$\begin{bmatrix} [M] & m \\ m\langle \chi \rangle & m \end{bmatrix} \begin{Bmatrix} \{\ddot{D}\} \\ l\ddot{\theta} \end{Bmatrix} + \begin{bmatrix} [C] & 0 \\ 0 & c \end{bmatrix} \begin{Bmatrix} \{\dot{D}\} \\ l\dot{\theta} \end{Bmatrix} + \begin{bmatrix} [K] & 0 \\ 0 & mg/l \end{bmatrix} \begin{Bmatrix} \{D\} \\ l\theta \end{Bmatrix} = \begin{Bmatrix} \{w(t)\} \\ 0 \end{Bmatrix} \quad \text{(5a-c)}$$

$$\langle \chi \rangle = \langle 1,0,0,0,\cdots,0 \rangle, \{w(t)\} = \langle w_1(t), w_2(t), ..., w_N(t) \rangle$$

where ($[M]$, $[C]$, $[K]$) are the mass, damping, stiffness matrices of the building structure, and $\{w(t)\}$ denote the along-wind load vector acting on the building. The wind loads acting on a building structure can be regarded as a random process with Davenport spectrum, and the cross power density of wind excitations is given by [3]

$$S_{ww,ij}(\omega) = K_D (2\rho C_D v_{10}^2)^2 A_i A_j \left(\frac{h_i h_j}{100} \right)^{\alpha} \frac{\eta_0^2}{|\omega|(1+\eta_0^2)^{4/3}} coh(\omega, h_i, h_j),$$

$$\eta_0 = \frac{L\omega}{2\pi v_{10}}, i,j = 1,2,3,...,N, coh(\omega,h_i,h_j) = \exp\left(\frac{-C_h |\omega| |h_i - h_j|}{2\pi v_{10}} \right) \quad \text{(6a-c)}$$

and K_D = surface coarse coefficient , ρ = density of air, C_D = drag coefficient, h_i = height of the ith floor, A_i = projection area of the ith story, $coh(\omega, h_i, h_j)$ = spatial coherence coefficient of the fluctuating along-wind forces, C_h = decay coefficient, L = wave length = 1200m, and v_{10} = mean wind velocity at 10m height. The complex frequency response matrix $[H(\omega)]$ can be given by

$$[H(\omega)] = \left[\begin{bmatrix} [K] & 0 \\ 0 & mg/l \end{bmatrix} + i\omega \begin{bmatrix} [C] & 0 \\ 0 & c \end{bmatrix} - \omega^2 \begin{bmatrix} [M] & m \\ m\langle \chi \rangle & m \end{bmatrix} \right]^{-1} \quad (7)$$

Therefore, the power density matrices of the floor displacements and velocity are given by the following equations, respectively

$$\left[S_{XX,ij}(\omega) \right] = \left[H(\omega) \right] \left[S_{w,ij}(\omega) \right] \left[H^*(\omega) \right]^T, \left[S_{\dot{X}\dot{X},ij}(\omega) \right] = \left[H(\omega) \right] \left[\omega^2 S_{w,ij}(\omega) \right] \left[H^*(\omega) \right]^T \quad \text{(8a,b)}$$

and the root mean square (RMS) of the displacement and velocity responses at the ith floor are respectively expressed as

$$\sigma_{X,i} = \sqrt{\int_0^\infty S_{XX,ii}(\omega) d\omega}, \sigma_{\dot{X},i} = \sqrt{\int_0^\infty S_{\dot{X}\dot{X},ii}(\omega) d\omega} \quad (9)$$

Table 1 Properties of the wind load and shear building

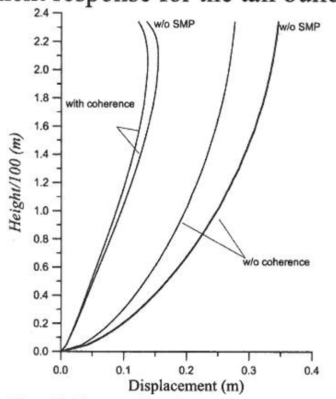
ρ (kg/m^3)	C_D	K_D	α	v_{10} (m/s)	m_i (t)	K (kN/m)	A_i (m^2)	h_i (m)	ω_1 (rad/s)	ω_2 (rad/s)	ω_3 (rad/s)	M_1(t)	M_2(t)	M_3(t)
1.28	1.2	0.02	0.19	15.5	248.5	$4.0*10^5$	182	4.5	1.20	3.6	6.0	6525	6536	6559

Table 2 optimal parameters of the suspended mass pendulum.

$\mu = m/M_1$	ω_t/ω_1	ζ
0.01	0.985	0.066

Numerical example

To illustrate the usefulness of the present mass pendulum for reducing the wind-excited vibration of tall buildings, a planar shear building with 52 stories is considered in this example. The structural height of this building is 234m. Let us assume that the shear building has the same lumped mass and flexural stiffness at each story and the height for each story of this building is 4.5m. The damping ratio of Rayleigh type for this structure is assumed to be 3%. The properties of this tall building and the parameters for the along-wind loading on the building are listed in Table 1. According to free vibration analysis, the first three vibration modes of this building are drawn in Fig. 3. and the dynamic characteristics have also been listed in Table 1. Let us select the tuning mass of the suspended mass pendulum to be 1% of the fundamental modal mass (M_1 = 6525t) of this building. To reduce the along wind-excited vibration of the building, the suspended mass pendulum with the optimum parameters listed in Table 2 for tuning the building response has been installed on the top floor, as depicted in Fig. 4. According to numerical calculations, Figs. 5 and 6 indicates that the installation of the mass pendulum results in significant suppression on the vibration responses of the tall building due to wind loads. On the other hand, the coherence effect of function (*coh*) shown in Eq. (6c) on the root mean square (RMS) of floor displacements and velocities demonstrates that the consideration of coherence function may result in significant difference on the estimation of displacement response for the tall building investigated herein.



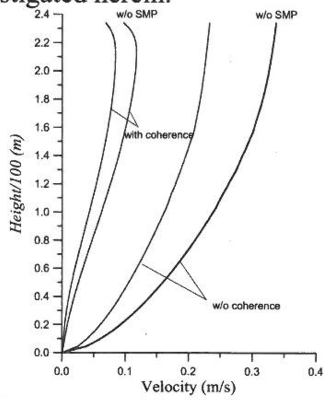


Fig. 5 RMS of floor displacement response. Fig. 6 RMS of floor velocity response.

Concluding remarks

In this paper, a suspended mass pendulum is demonstrated to be a useful passive vibration control device for reducing the wind-induced response of a tall building. By modeling the tall building as a shear building, and the suspended mass pendulum as a lumped mass hanged by a mass-less wire, one can investigate the dynamic response of the building under the action of wind forces determined by Davenport spectrum. From the numerical example, it shows that the control effectiveness for displacement response of the tall building with 52 stories by utilizing a mass pendulum is good. Moreover, if the coherence function of the wind loads based on Davenport spectrum is neglected; the calculated results indicate that the response of the building would become conservative, which agrees to the safe consideration in wind-resistance design for buildings.

Acknowledgements

This study was sponsored in part by the 2005 integrated research project of *Wind Measurements and Wind-Induced Vibrations of Tall Buildings* in Tamkang University, such a financial support is gratefully acknowledged.

References

1. Leipholz, H.H.E. and Abdel-Rohman, M. (1986) *Control of Structures*, Martinus Nijhoff Publishers.
2. Li, Q.S., Cao, H., Li, G., Li, S., and Liu, D. (1999) "Optimal design of wind-induced vibration control of tall building and high-rise structures," *Wind & Structures*, 2(1), 69-83.
3. Ying, Z.G., Ni, Y.Q., and Ko, J.M. (2005) "Semi-active control of linearized systems with multi-degree of freedom and application," *J. Sound and Vibration*, 279, 373-388.
4. Randall, S. E., Halsted, D. M., and Taylor, D. L. (1981) "Optimum vibration absorbers for linear damped systems," *ASME Journal of Mechanical Design*, 103, 908-913.