The role of non-linear Damping in response estimation

by

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INTRODUCTION

The damping parameter is a measure of the rate at which energy that acts on a structure is dissipated. The damping force acts to reduce the response at every instant throughout each cycle of response of the structure. For this reason the correct choice of a value for damping is of crucial importance for the estimation of the actual response of structures. This is especially so if wind tunnel tests have been conducted to ascertain the generalised response of a proposed design. The superposition of either a mathematical or imposed mechanical damping value will alter the estimate of response fundamentally.

More recently the non linear nature of the damping parameter has been identified and quantified [1]. The values of damping start at small values and rise as the amplitude of response increases. It has been assumed that if the value of damping at the maximum response of interest is chosen, then the estimate of the overall response will be accurate.

It is possible to model damping as a fixed value, or as a value that increases with increasing amplitude. In this paper a comparison is made of the predicted response using these two approaches. The difference in the predicted response is much larger than had been previously supposed, and in the example given here, the maximum response is approximately 100% greater under the assumption of a non-linear damping

1. Damping In The Equations Of Motion

Damping has traditionally been treated as being proportional to velocity. The implication of this is that we have viscous damping as the mechanism. Many people have become convinced that viscous damping really exists, but in practice it is a mathematical convenience. The equations become simpler and easier to handle if that is what we assume. It is generally forgotten that when the equations of motion were popularised, the concept of 'equivalent viscous damping' was also introduced [2].

Despite the fact that the mechanism of damping has now positively been identified as being caused by friction (and therefore proportional to displacement) there is a detail of the behaviour that re-establishes the validity of the concept of equivalent viscous damping [3]. If an assumption of viscous damping is made then the relationship between the force created by an action and the response of a structure, can be expressed as:

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$$x_r = F_r\left[\frac{1/K_r}{1-\frac{f^2}{f_r^2}+j2\zeta_r\frac{f}{f_r}}\right]\text{..........}(1)$$

In which K_r is the stiffness associated with mode 'r', f_r is the resonance frequency of that mode and F_r is the force acting on that mode and creating the response x_r in that mode.

2. Run-down method of Damping Measurement

In this methodology the structure is artificially shaken at a resonance frequency. The resulting vibration is allowed to build up to a steady state sinusoidal response, and then the excitation source is suddenly stopped. The resulting response is of the form shown in Fig 3. The basic assumption is that if the values of damping and the resonance frequency are constant values, then the resulting decay of oscillation will follow the form shown by the inner trace in figure 3. However, if the damping varies with amplitude then the resulting response will be larger and is shown by the outer trace in figure 3. This factor is of great importance if the peak response for an extreme even is required. Val and Segal [4] have recently pointed out that this can cause errors of as large as 30% in the estimation of displacement, and this can translate to a 200% error in the calculation of stresses.

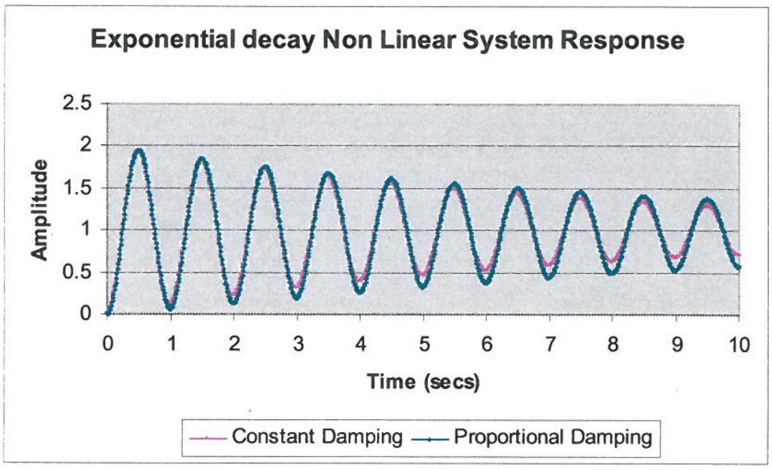

Fig 3: Decay of oscillation of a linear single degree of freedom system.

If the peak values of each cycle of oscillation are plotted against the cycle number, then the resultant plot shows a straight line for a linear system. This demonstrates that the envelope of the decay has the form of an exponential decay of the form $e^{-\zeta\omega_r t}$. Fig 4 shows such a plot taken from the response of a building after steady state induced vibration had been suddenly stopped. If the value of ζ were constant then the curve would become a straight line.

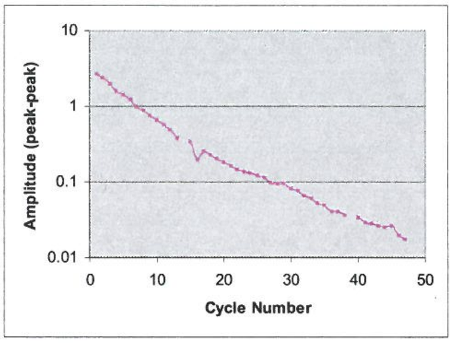

Fig 4: Peak values against cycle number for a prototype building

In practice, there are non-linearities of both the resonance frequency and of the damping [6,7]. Induced vibration tests conducted on full scale buildings first established this fact under well controlled forced vibration tests undertaken in the 1980's in the UK[5]. In the example shown in Fig 4 it is apparent that with encroaching lower amplitudes, the value of damping decreases until the point at which it assumes a constant value (where the envelope of peak values becomes essentially a straight line).

3. Theoretical Simulation of Response

Using a damping characteristic similar to that shown in Fig 4 (and to those suggested by Jeary[8], Lagomarsino [9] and Tamura [10]) a response of a structure to an imposed load was calculated both for the constant damping assumption (the design value), and for the proportional characteristic. The input function (the modal force created by wind action) was simulated by the time series shown in Fig 5.

The important aspect of this selected modal force input is that a gust, having a significant amount of energy at the resonance frequency of the structure, arrives in-phase with a pre-existing motion of the structure.

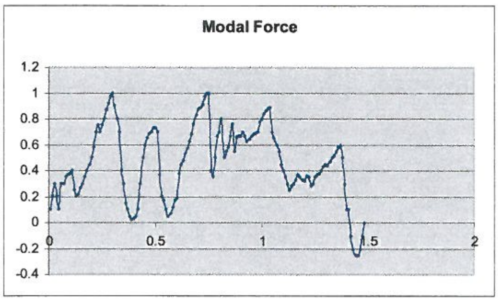

Fig 5 Modal Force simulation

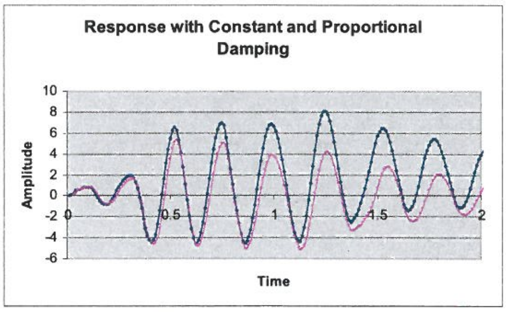

Fig 6 Modal Response with Constant Damping (smaller trace) and Proportional Damping (Larger Trace)

The simulation of response was conducted using a single degree of freedom system and an artificially selected sequence of wind pressure variations, purely for the purpose of demonstrating the differing effects caused by different assumptions of the damping characteristic. The smaller trace used a constant (design) value of damping of 0.5%, whilst the proportional damping was simulated using a low amplitude value of 0.1% and then an increasing value up to a maximum of 0.5%. The maximum values of damping are the same for both cases.

The differences in the two traces are therefore caused only by the lower damping values associated with the time the structure spends at lower amplitudes. In other words during each cycle of response there is a low amplitude part of the cycle during which the dissipation of energy is lower than those values that occur at the peak of the cycle.

Conclusion and Recommendation

It has been shown that errors in response calculations, that may be as large as 100%, can be generated by assuming that the maximum (or design) value of damping applies at all amplitudes through each cycle of response. As a result of this result it is recommended that the effects of non-linear damping should be considered in addition to other methods (such as wind tunnel testing), when an evaluation of the maximum likely response of a structure is considered.

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