# Comparison of dynamic characteristics from full-scale measurements and finite element models of a tall building

Alan Moore and Graeme Wood
Wind Engineering Services, Department of Civil Engineering, The University of Sydney

2 February 2006

Abstract: This report describes a comparison between measured and predicted natural frequencies for a 190 metre tall office building. The measured natural frequencies are determined by a full-scale forced vibration test using a mechanical shaker. The predicted natural natural frequencies are attained using finite element analysis of the structure. A number of finite element model iterations are compared, with each iteration focusing on a particular aspect of the structure to attain more knowledge regarding the structures behaviour. Particular attention is directed toward the loading rate of concrete and its effect on the elastic modulus of concrete. It is suggested that structures with shorter periods of vibration that ignore the loading rate effects may consistently underestimate the natural frequencies when using finite element models.

#### 1 Introduction

Structural engineers regularly use finite element analysis to facilitate the design of tall buildings. The finite element models are used to predict the dynamic characteristics of the entire structure, as well as to determine loads on the individual structural elements. Accurate predictions of the dynamic characteristics, including the natural frequencies and mode shapes, are important parameters for use in wind tunnel testing to determine design wind loads and the wind-induced responses.

Despite advances in finite element analysis of tall buildings, discrepancies between the predicted and actual dynamic characteristics are still encountered. Since the actual dynamic characteristics cannot be accurately measured until the structure is almost completed, there is a reliance on finite element models to predict dynamic characteristics to avoid costly alterations to the structure, such as the installation of damping devices. By comparing finite element models and full-scale measurements, it is possible to improve understanding of the mechanisms that affect the dynamic behaviour of tall buildings and to build a useful database of knowledge regarding finite element modelling of tall buildings.

A number of similar studies on finite element predictions of natural frequencies [1] [3] have reported lower predicted natural frequencies compared to measured values. In the case of Li et. al. 2004, the underestimation was approximately 20% for a 300 metre tall office building. The difference was attributed to either an over-estimation of mass, or an under-estimation of stiffness due to the omission of non-structural components (block work and in-fill walls) in the finite element model. This article didn't mention the concrete modulus of elasticity used in the finite element modelling.

# 2 Description of Structure

The building investigated is a 46 storey office tower located in the Sydney CBD, with a maximum height of approximately 190 m above street level and 20 m of underground levels. The floor plan is approximately rectangular, with small rectangular sections of the floor plan removed in the north-east and south-west corners of the tower. A composite structural system was used for the design, consisting of reinforced concrete and structural steel components. The centrally located core is constructed of reinforced concrete. Steel beams and a composite slab span from the core to the perimeter columns, which consist of concrete filled steel hollow sections.

The building also includes a secondary core that connects to the rear face of the main core. This secondary core starts at level 20, and therefore does not connect to the foundations. In this configuration, the secondary core 'piggy-backs'

the main core and creates an eccentricity between the centre of mass and the centre of stiffness. Block work in-fill is used at the core, but is too minimal to have any significant influence on the structural system.

Construction of the tower utilises an existing reinforced concrete structure that was partially completed and then abandoned ten years prior. The existing core was completed to level 23, and the floors to level 14. As the tower construction progressed upwards, a substantial amount of demolition work to remove existing floor slabs and core sections was completed simultaneously. The demolished floor slabs and core sections were re-constructed according to revised designs and alignments.

The major axes of the building correspond to the cardinal points, and are displayed in Figure 1.

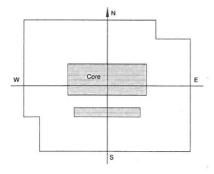


Figure 1: Plan view of tested building with major axes and orientation.

### 3 Forced Vibration Testing

Natural frequencies and mode shapes were determined for the first three modes of vibration using a forced vibration test. Levels 16, 34 and 51 were used to determine the mode shapes, with three accelerometers located at each level - two at the core to measure the NS and EW accelerations and one positioned at the western face to measure the torsional accelerations.

A mechanical shaker with a mass of approximately 1,000 kg was used to excite the building. The shaker is electronically controlled and generates the excitation force by moving the mass in a sinusoidal manner. A ball-screw mechanism smoothly moves the mass at a specified rate and enables instantaneous stopping of the mass for

accurate damping detection.

To reduce the testing time, all nine accelerometers were connected via three computers and an ethernet network. This allowed simultaneous measurement of accelerations at all three levels. Previous methods have used a reference and a rover accelerometer approach, which requires the rover to be moved numerous times per mode shape test.

The natural frequencies for the first three modes of vibration are included in Table 1 along with the results of the finite element analysis. Also included in Table 1 are the damping values and corresponding acceleration amplitudes for each of the three modes tested. The mode shape for the first mode of vibration acquired from forced vibration testing is compared to the mode shape attained from the finite element model in Figure 2.

### 4 Finite Element Analysis

Instead of generating the finite element model from scratch, the model used by the designers of the building was taken as the starting point. Using the designers model was appropriate for this research because one of the aims is to determine why there are discrepancies between the predicted and full-scale values. The model from the designers (FEM1) included all of the structural elements that are critical for a dynamic analysis — the reinforced concrete core with elevator and service openings, perimeter columns, floor slabs and outrigger trusses. The appropriate beam end releases were also included in the model, as well as translational masses for steel and plant equipment.

The initial mofel was compiled in Strand7, a commercial finite element package, and this software was used for subsequent analysis. Table 1 displays the natural frequencies from the finite element analysis along with the natural frequencies from full-scale vibration. The natural frequencies from FEM1 are underestimating the measured values by almost 30%, which is a significant difference. A large portion of this initial discrepancy was attributed to both the foundation restraints and the stiffness values used in the model.

The foundation restraints for FEM1 only modelled level 1 as being fixed, despite levels 1 to 9 being below street level. Given the sandstone and concrete from neighbouring structures surrounding the underground levels, the perimeter nodes of levels 1 to 9 were fixed. The model FEM2 in-

cludes these changes, and with these changes still underestimates the natural frequencies by 16%.

Adjusting the stiffness of a structure can have significant impacts on its natural frequencies. Unfortunately, in the case of reinforced concrete structures, accurately determining the modulus of elasticity of concrete is difficult and highly variable. The variability depends on quality of the supplied concrete, the age of the concrete, the rate of loading as well as other factors.

The specified concrete strength for the core sections of the test structure was  $f_c'=50$  MPa. The expected strength of the supplied concrete is likely to be higher than specified, as suppliers are required to ensure no more than 5% of the concrete delivered is below specification. This results in expected compressive strengths closer to  $f_c'=55$  MPa. Furthermore, these strength values are for concrete at 28 days, which needs to be adjusted by a factor that accounts for strength increases as the concrete ages. After one year, the Cement and Concrete Association of Australia recommends a factor of 1.3 for normal portland cement.

The rate of loading also influences the compressive strength of concrete, resulting in higher reported strengths when concrete is loaded at a faster rate than specified in standard compressive strength tests. Mirza et. al. [2] proposed the following equation to determine the effect of loading rates on the expected elastic modulus of concrete.

$$E_{cR} = (1.16 - 0.08 log t) E_c \tag{1}$$

The parameter t is the loading duration in seconds. This correction is applicable to the test building due to the dynamic nature of the loading. The first mode period of vibration for the test structure is approximately four seconds, resulting in a one second loading duration. Cracking of the core sections was not included in the stiffness estimates due to the continuous compression force applied to core elements, and the lack of prior significant events to cause cracking of the core walls.

Applying the above factors and corrections to the specified concrete for the test structure results in an elastic modulus of approximately 4,700 MPa, which is significantly higher than the 3,800 MPa used in FEM1. This elastic modulus adjustment is included in FEM3, and Table 1 indicates the adjusted elastic modulus value has considerably improved the natural frequency predictions. However, the natural frequency for the torsional

mode of vibration is still significantly different to the measured value.

Accurate modelling of the core header beams can have a big impact on the torsional vibration [3]. After modelling the core header beams to more accurately represent their strength and geometry, a fourth and final model was compiled. This model, labeled FEM4, has significantly improved torsional vibration comparisons at the expense of the EW direction overestimating the natural frequency. It is possible that the final discrepancies in the finite element model could be attributed to an overestimation of the mass in the upper levels. However, the values for the lumped masses for steel reinforcement appeared correct for the structures design.

	NS	EW	Torsion
Measured (Hz)	0.25	0.29	0.41
FEM1 (Hz)	0.18	0.22	0.30
FEM2 (Hz)	0.21	0.26	0.35
FEM3 (Hz)	0.23	0.29	0.33
FEM4 (Hz)	0.24	0.31	0.40
Damping	0.91%	0.71%	0.92%
Accel (milli-g)	0.2	0.1	0.3

Table 1: Measured and predicted natural frequencies.

#### 5 Discussion

From the low elastic modulus values used in FEM1, it would appear that this initial model was setup by the design engineer to investigate structural distortions from concrete creep. By using such low modulus values at key points within the core, the model was tuned for such an analysis. However, a model tuned for a worst case senario concrete creep analysis is not able to model the wind-induced dynamic loading effects.

To model dynamic loading of concrete, both the expected aged strength and the loading rate effects need to be accounted for. These two factors can have a significant impact on the concrete stiffness - in the case of loading rate factor by as much as 116% [2]. FEM4 accounts for these two factors, and achieves more comparable results for the predicted natural frequencies compared to the previous model iterations.

If the dynamic loading of concrete is indeed a key factor in determining the stiffness of concrete for dynamic loading, this would lead to possible underestimation of natural frequencies in finite element models of structures with higher natural frequencies. Taller buildings have longer dynamic loading periods, and therefore their concrete stiffness is not greatly influenced by dynamic loading. This concept is apparent in Equation 1, since a value of t=1 second results in a 116% increase in the expected elastic modulus of concrete. If the dynamic loading effects are not included in FE models, then the predicted natural frequencies will be lower than prototype values. It is therefore expected that buildings bellow approximately 200 metres will be more susceptible to these errors, and their finite element models will underestimate natural frequencies.

Figure 2 displays partial correlation between the predicted and measured mode shapes. The torsion mode shapes are least in agreement, and this could be due to incorrect modelling of the inplane rigid actions of the floor slabs. Modelling the slabs as rigid diphragms was achieved be creating inter-nodal constraints in the plane of the floor slabs. However, due to the inherent complexity of finite element models, it is possible that the rigid diaphragm assumption does not hold for the entire structure.

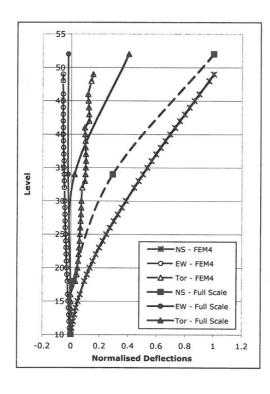


Figure 2: Comparison of full-scale and FEM4 mode shape for the first mode of vibration.

### 6 Conclusions

By using a comparison between measured and predicted natural frequencies for a 190 metre tall office building, it is observed that further understanding of the mechanisms that affect the dynamic behaviour of tall buildings is achievable. The initial finite element model from the design engineer underestimated the natural frequencies by approximately 30%. Successive iterations of the model focused on improving the modelling of foundations, concrete stiffness and core header beams.

By accounting for the loading rate effects on concrete stiffness due to dynamic loading, a higher elastic modulus was estimated and used for sections of the core where much lower elastic modulus values had been initially specified. It is suggested that these lower values were used to determine structural distortions from concrete creep effects, and they represent a worse case scenario in this case. In addition, by ignoring the loading rate effects on concrete stiffness it is possible that structures with shorter periods of vibration may consistently underestimate the natural frequencies when using finite element models.

## 7 Acknowledgments

The authors thank Andy Davids, Greg Morris and Kabilan Subramaniam from Hyder Consulting and Harry Young from Multiplex for their assistance in undertaking the numerous full-scale tests.

#### References

- [1] Q.S. Li and J.R. Wu. Correlation of dynamic characteristics of a super-tall building from full-scale measurements and numerical analysis with various finite element models. *Earth-quake Engineering and Structural Dynamics*, 33(14):1311–1336, November 2004.
- [2] S.A. Mirza, M. Hatzinikolas, and J.G. Mac-Gregor. Statistical descriptions of strength of concrete. *Journal of the Structural Division*, ST6:1021–1037, June 1979.
- [3] Carlos E. Ventura and Norman D. Schuster. Structural dynamic properties of a reinforced concrete high-rise building during construction. *Canadian Journal of Civil Engineering*, 23(4):950–972, August 1996.