Bi-Modal Weibull Distribution Fit for Climatic Wind Speed Histogram

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ABSTRACT A bi-modal Weibull distribution fitting technique is presented which can be used for climate model development in wind engineering applications. Its use is advantageous in situations where more than one type of weather phenomena influences wind speed statistics. The approach described in this paper was developed at the Boundary Layer Wind Tunnel Laboratory (BLWTL) at the University of Western Ontario. The methodology and programming criteria are addressed. The method has more flexibility and higher accuracy by about one order of magnitude compared with traditional Weibull fits.

Introduction

The Weibull distribution has been used by the wind engineering community as a model for describing the distribution of wind speed and direction [1] for almost 40 years. The model can be described as follows.

Assuming that the probability of exceeding a certain speed V follows the Weibull distribution:

$$P(>V) = e^{-(\frac{V}{c})^k} \quad (1)$$

where c is a scaling parameter and k is the shape parameter, then the probability density can be expressed by

$$p(V) = \frac{k}{c}(\frac{V}{c})^{k-1} e^{-(\frac{V}{c})^k} \quad (2)$$

In connection with the wind engineering design of a tall tower in Dubai, Feng and Irwin [2] developed a "double" Weibull model to describe surface level wind data from Dubai which, in this particular case, is poorly fitted by the conventional single Weibull model.

In this "double' Weibull model, five parameters, c_L, k_L, V_{thr}, c_U and k_U, were estimated by:

$$P(>V) = e^{-(\frac{V}{c_L})^{k_L}}, for V <= V_{thr} \quad (3a)$$

$$P(>V) = e^{-(\frac{V}{c_U})^{k_U}}, for V > V_{thr} \quad (3b)$$

where, V_{thr} is a threshold velocity which separates the wind data into the two speed regions (upper and lower regions). In each region, a corresponding single Weibull distribution is used.

To determine the best overall fit using the Feng and Irwin approach, the c and k for the upper and lower wind speed ranges are first calculated for a full range of threshold velocities, using a minimum number of 3 data points in each of the upper and lower wind speed ranges.

To determine the optimum fitting, the goodness of fit is calculated for both the high and the low fits for each set of parameters associated with all possible threshold velocities (V_{thr}). This is calculated using the χ^2 parameters as:

$$\chi^2 = \sum_j \frac{(n_j - \hat{n}_j)^2}{\hat{n}_j} \qquad (4)$$

where n_j is the number of observations in wind speed bin j and $\hat{n}_j$ is the calculated, or expected number of observations in wind speed bin j determined from the Weibull parameters.

The lower and upper c's and k's, as well as the V_{thr} parameters selected are those corresponding to the fits where $| \chi^2_{upper} - \chi^2_{lower} |$ is a minimum.

This "double" Weibull fit introduces three additional parameters which provide more flexibility in fitting. However the use of a discrete threshold speed as one of the parameters introduces a discontinuity in the mathematical model.

In this paper, a bi-modal Weibull distribution is presented. The model has a continuous distribution and can provide a better fit than the Feng and Irwin method.

Theory

Assuming that for a certain wind direction the total probability of exceeding a certain wind speed V follows a linear combination of two Weibull distributions, called the bi-modal Weibull distribution. This distribution can be written as follows:

$$P_{(bi-fit)}(>V) = w_1 e^{-(\frac{V}{c_1})^{k_1}} + (1-w_1) e^{-(\frac{V}{c_2})^{k_2}} \qquad (5)$$

where the probability density of the winds will then be described by:

$$p(V) = w_1 \frac{k_1}{c_1}(\frac{V}{c_1})^{k_1-1} e^{-(\frac{V}{c_1})^{k_1}} + (1-w_1)\frac{k_2}{c_2}(\frac{V}{c_2})^{k_2-1} e^{-(\frac{V}{c_2})^{k_2}} \qquad (6)$$

where w_1 and $1\text{-}w_1$ are the weighting factors of the corresponding distributions. The second weighting factor, $(1\text{-}w_1)$, can also be called w_2, such that $w_1+w_2=1$. This is consistent with the general definition used in multi-modal distributions[3] .

Optimal estimates of the parameters of this distribution can be made using the least-squared error technique, described in the next section. In wind engineering applications for the prediction of the extreme wind speeds, wind loads or responses, the probability density is integrated over all wind directions, α , to calculate the crossing rate as follows:

$$N_X(X_1) = \sqrt{2\pi}\nu\sigma \int_0^{2\pi} \{1 + (\frac{dV_1}{V_1 d\alpha})^2\}^{0.5} p_V(V,\alpha) d\alpha \qquad (7)$$

Combining Eq. 6 and Eq. 7, calculation of the crossing rate of a particular response boundary $X_1=X(V_1, \alpha)$, is computed as follows:

$$N_X(X_1) = \sqrt{2\pi}\nu \int_0^{2\pi} \{1 + (\frac{dV_1}{V_1 d\alpha})^2\}^{0.5} A(\alpha)\sigma_{bi} [w_1 \frac{k_1}{c_1}(\frac{V}{c_1})^{k_1-1} e^{-(\frac{V}{c_1})^{k_1}} + (1-w_1)\frac{k_2}{c_2}(\frac{V}{c_2})^{k_2-1} e^{-(\frac{V}{c_2})^{k_2}}] d\alpha \qquad (8)$$

where the various w, k, c parameters are defined for each corresponding direction α ; ν is the cycling rate of wind; and the standard deviation σ_{bi} is expressed by:

$$\sigma_{bi} = \sqrt{w_1\sigma_1^2 + w_2\sigma_2^2 + w_1w_2(\mu_1 - \mu_2)^2} \quad (9)$$

where μ's are mean values for the corresponding Weibull distributions.

Implementation

To determine the five parameters, an optimization technique is used based on non-linear programming search. The physical meaning of the parameters is the same as that in a single Weibull distribution: k represents the shape factor and c is a scaling factor for wind speed. To obtain these parameters, a two-step parameter-solving method is used.

Firstly the uni-modal fit with the Log-scale-error-minimization is used to obtain the parameter k_1 for the higher speed range, and the uni-modal fit with the Linear-scale-error-minimization is used to obtain k_2 for the lower speed range; i.e.

$$\min E^2 = \min \sum_1^N (\ln P_{uni\_fit}(>V) - \ln P_i)^2 \text{ to determine } k_1, \quad (10)$$

$$\min E^2 = \min \sum_1^N (P_{uni\_fit}(>V) - P_i)^2 \text{ to determine } k_2, \quad (11)$$

The following criterion is then used to determine the parameters c_1, c_2 and w_1:

$$\min E^2 = \min \sum_1^N \frac{(P_{(bi\_fit)} - P_i)^2}{(P_{(bi\_fit)} + P_i)^{1.75}} \quad (12)$$

A comparison of the various fitting techniques for a particular set of data is shown in Figure 1 and an accuracy comparison is provided in Table 1. It can be seen that by using the bi-modal approach, the fit accuracy has improved and the total error has decreased by almost one order of magnitude when compared with uni-modal fit and the "double" Weibull model.

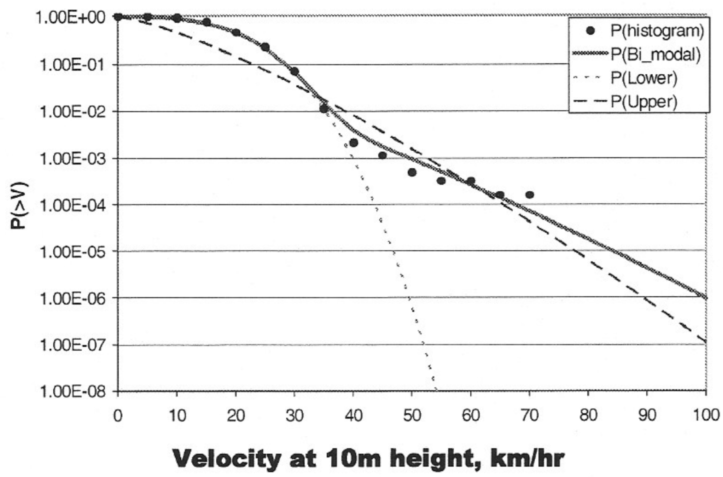


Figure 1 Distribution fit comparison: P(Lower) – uni-modal fit with linear-scale error, P(Upper) – uni-modal fit with Log-scale error and P(Bi-modal) – two-step fit.

Table 1 Fit accuracy comparison for a particular wind histogram

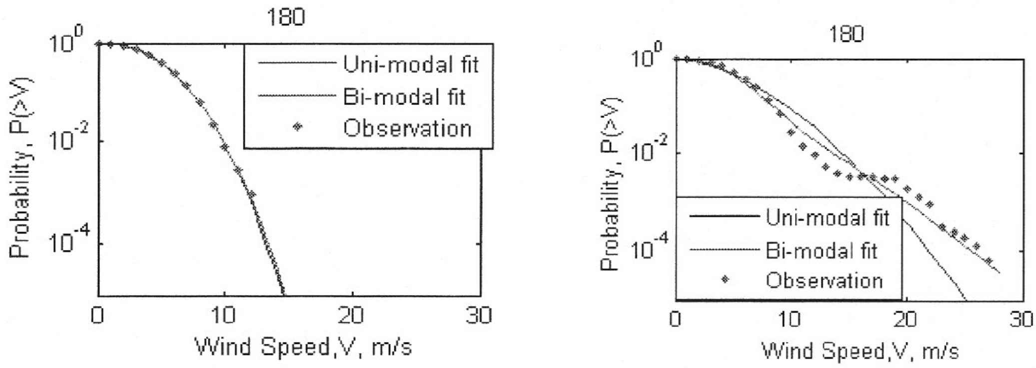
	Uni-modal fit	"Double" Weibull[2]	Bi-modal fit
Total squared errors	0.005876	0.003345	0.000618

The bi-modal fitting method was developed at the BLWTL at the University of Western Ontario[4]. It is now also being used at Vipac.

Discussion

There is sometimes a need to improve the underlying model and the fitting method for some particular wind histogram data, such as those from Dubai. The bi-modal fitting method presented here has a high degree of fit accuracy and maintains the same physical meanings of the fitting parameters. The bi-modal fitting method provides added improvement for statistically describing data which can not be fitted adequately by the single Weibull model. The bi-modal fitting method has very good flexibility and is a suitable fitting method for climate model development in wind engineering applications.

The uni-modal and bi-modal fitting methods have been used for two stations in Australia: Melbourne (Station #086086) and Queensland (Station # 031037). The uni- and bi-modal fits for the two stations for 180° azimuth are shown in Figures 2a and 2b. It can be seen that in the case of the Melbourne station, the two fitting methods are comparable while for the Queensland station, the two fitting methods show substantial difference, especially at higher wind speeds. The bi-modal fit, however, provides a better fit to the Queensland data.



a) b)

Figure 2 Weibull fits for 180° azimuth angle for two stations in Australia
a) Melbourne Int. Airport (Station # 086286), b) Low Isles Lighthouse, Queensland (Station # 031037).

References

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