The Damage Potential of Windborne Debris

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1 INTRODUCTION

The damage caused by windborne debris is a complex function of the wind conditions, the availability of debris, the point of release, the aerodynamic characteristics of the debris, the impact dynamics and the strength of the structure impacted. This paper will primarily focus on the aerodynamic aspects of this problem and their relationship to the other parts of the problem. Holmes [1] in reviewing damage risk models for windborne debris comments "The damage produced by an impacting missile on a building surface is dependent on the component of momentum normal to the surface and/or its kinetic energy at impact". He also notes that tests at James Cook University [2] showed that rod-type missile damage of spaced fibreboards indicated better correlation with impact kinetic energy than with impact momentum. Since various building materials, including wood, behave almost elastically up to the point of sudden failure Richards et al. [3] modelled, both numerically and in simple laboratory experiments, the impact of rod-type debris against an elastic vertical surface under various conditions. It can be shown that under such conditions the maximum possible force is given by

$$F_{Max} = V\sqrt{mk} \tag{1}$$

where V is the magnitude of the total velocity at impact and m the mass of the object and k is the stiffness of the impacted surface in N/m displacement normal to the surface. It is shown that with oblique impacts, where the velocity vector is at an angle θ_w relative to the surface normal, the observed and modelled peak forces decrease approximately with $\cos(\theta_w)$, giving

$$F_{Peak} \approx V\cos(\theta_w)\sqrt{mk} \approx V_{Normal}\sqrt{mk} \tag{2}$$

Hence for impacts against a vertical wall normal to the wind, the expected peak force is related to the horizontal velocity component u or the kinetic energy based on the horizontal speed ($E=mu^2/2$) by

$$F_{Peak} \approx u\sqrt{mk} \approx \sqrt{2Ek} \tag{3}$$

Simple logic suggests that if an object is very light for its size it will accelerate quickly and travel a long way but even then it will still only have limited kinetic energy and is hence unlikely to cause significant damage. On the other hand a very heavy object will fall quickly to the ground and will only slowly gain horizontal speed and hence is unlikely to cause a damaging impact. This paper will investigate the middle ground and seek to determine what objects are most likely to cause damage.

2 TRAJECTORY MODELLING

While the flight of sheet and rod type debris is more complex than compact debris, the work of Lin et al. [4, 5] clearly shows that the ensemble behaviour is similar in character to that of a compact object with the same average drag coefficient. The details of a particular flight simply cause scatter around the general trends. Some conclusions regarding the likely behaviour of a particular object may therefore be drawn from analysis of the compact debris equations. The acceleration equations for a compact object in a uniform horizontal wind of speed U, as noted by Holmes [6], are

$$\frac{du}{dt} = B(U-u)\sqrt{(U-u)^2 + v^2} \quad (4)$$

$$\frac{dv}{dt} = -g + B(-v)\sqrt{(U-u)^2 + v^2} \quad (5)$$

where u and v are the horizontal and vertical velocity components of the object respectively and g is the acceleration due to gravity. The variable B is referred to as the ballistic coefficient:

$$B = \frac{C_D \rho A}{2m}, \quad (6)$$

where C_D is the drag coefficient, ρ the air density, A the cross-sectional area and m the mass. In this form the ballistic coefficient characterises the significance of aerodynamic drag in relation to the associated acceleration. It may be noted that there are other forms of the ballistic coefficient including m/C_DA, which is related to the inverse of that used here.

Lin et al. [5] note that in the very early stages of the flight, assuming zero initial velocity, then

$$u = BU^2 t \quad \text{and} \quad v = gt, \quad (7)$$

whereas for large times Baker [7] gives the asymptotic velocities as:

$$u = U \quad \text{and} \quad v = -\sqrt{\frac{g}{B}}\text{, the terminal velocity.} \quad (8)$$

While these limiting cases should be kept in mind, what is sought here are approximate solutions which are valid over moderate periods of time. It may be noted that initially both u and v are small and so while $(U\text{-}u)>>v$ then Eq. (4) may be approximated as

$$\frac{du}{dt} \approx k(U-u)^2 \quad (9)$$

This, as noted by Holmes [6], can be integrated by using separation of variables to give

$$u = U\left(\frac{BUt}{1+BUt}\right), \quad (10)$$

which in the limits matches both Eq. 7(a) and 8(a). Integration of Eq. (10), with the initial condition that x=0 at t=0, yields

$$x = Ut - \frac{\ln(1+BUt)}{B} \quad (11)$$

Both Baker [7] and Lin et al. [4, 5] suggest that the relationship between the horizontal velocity and the horizontal distance can be well represented by a function of the form:

$$u = U\left(1 - \exp\left(-\sqrt{2Bx}\right)\right) \quad (12)$$

Lin et al. [4, 5] also suggest that the distance-time relationship can be modelled by a polynomial, which can be rearranged into the form:

$$x = \frac{1}{2}BU^2t^2 + \frac{aC_D}{B}\left(\frac{B}{C_D}Ut\right)^3 + \frac{bC_D}{B}\left(\frac{B}{C_D}Ut\right)^4 + \frac{cC_D}{B}\left(\frac{B}{C_D}Ut\right)^5 + \ldots\ldots \quad (13)$$

Fig. 1 shows horizontal distance and velocity curves for the 0.54g, 8mm diameter stone sphere investigated by Holmes [6] in a 20 m/s wind. The drag coefficient for a sphere at these Reynolds numbers is approximately 0.5 and so the ballistic coefficient B=0.235. Fig. 1(a) shows the horizontal velocity-distance relationships given by numerical integration of the equations of motion, by combining Eq. (10) and (11) and the Baker/Lin et al. formula Eq. (12). While all three curves are similar Eq. (10) does underestimate the velocity at high distances where the vertical velocity *v* is of the same order or larger than the horizontal relative velocity (U-u) and so the approximation of Eq. (9) is no longer valid. Fig. 1(b) shows the same data plotted against time. Here the major limitation of Eq. (13) is apparent since the curve becomes unrealistic beyond times of 1s. However it may be recognised that the empirical data fitted by Lin et al. [5] to give C_D=0.496, a=0.084, b=-0.1 and c=0.006 only covered a limited time range which in this case is effectively from 0-0.54s and so use of Eq. (13) outside this range is unadvisable. Even if the coefficients could be refined with longer duration data it is almost inevitable that polynomials will not give sensible larger time behaviours whereas Eq. (11) only slightly underestimates the numerical values.

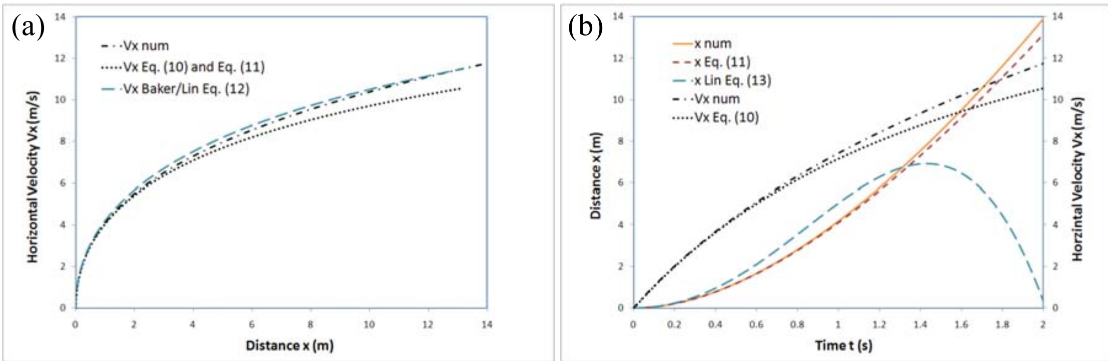

Fig. 1. Horizontal distance and speed of a 0.54g, 8mm diameter spherical stone in a 20 m/s wind, (a) speed as a function of distance and (b) both distance and speed as a function of time.

The vertical acceleration equation (Eq. (5)) can also be approximated by noting that in the early stages the gravitational acceleration dominates and that it is only as the vertical velocity increases that the second term becomes significant, by which time the horizontal relative velocity (U-u) has reduced considerably. Hence, assuming that $v<0$,

$$\frac{dv}{dt} \approx -g + B(v)^2 \tag{14}$$

which may be solved by separation of variable to give

$$v = -\sqrt{\frac{g}{B}} \frac{\left(1 - e^{-2t\sqrt{gB}}\right)}{\left(1 + e^{-2t\sqrt{gB}}\right)} \tag{15}$$

At low and high times this matches 7(b) and 8(b) respectively. Further integration, with the initial condition $y(0)=y_0$, gives

$$y = y_0 - \sqrt{\frac{g}{B}} t - \frac{1}{B} \ln\left(\frac{1 + e^{-2t\sqrt{gB}}}{2}\right) \tag{16}$$

In order to be useful in the current analysis it is desirable to rearrange Eq. (16) in order to find the time at which the object has fallen a set distance. This may be simplified by noting that the exponential term is only significant at low times when the vertical motion is dominated by gravity, resulting in

$$t \approx \sqrt{\frac{B}{g}}(y_0 - y) - \frac{1}{\sqrt{gB}} \ln\left(\frac{1 + e^{-2\sqrt{2B(y_0 - y)}}}{2}\right) \tag{17}$$

3 IMPACT ENERGIES

Equations (10), (11) and (17) have been used to determine the likely impact kinetic energies (E) of a range of projectiles with the release points at various heights above the point of impact in a 20m/s uniform wind. The base projectile considered was the 2.4 x 0.1 x 0.05m rod which Lin et al. [5] found had an average drag coefficient of 0.8 (based on the largest face area). This standard wooden test missile normally has a mass of 4 kg but in this case a range of masses were considered ranging from 0.5 kg up to 90 kg, which represent a range of materials from poly-styrene to steel. Fig. 2 shows a contour map of the resulting impact kinetic energies.

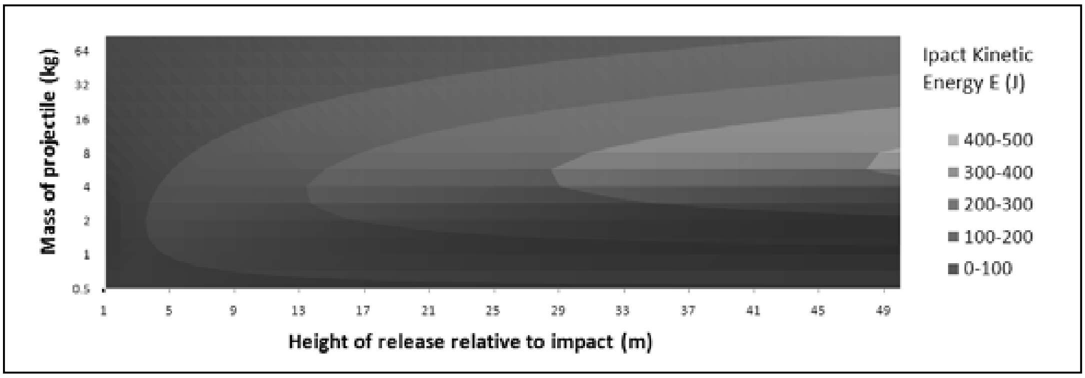

Fig. 2. Impact kinetic energies for a range of projectile when falling from specific heights.

This analysis shows that the lightest (poly-styrene) projectile can travel a long way, up to 150m if it originates from 50m above the impact point, and rapidly achieves high proportions of the wind speed but will never exceed 100J of energy. At the other extreme the heaviest (steel) projectile falls to the ground within 2.5m horizontally, even from a 50m height, and also barely exceeds 100J of kinetic energy based on horizontal speed. However between these extremes higher energies do occur. If the points or origin for debris are all below 5m then a 2 kg rod has the highest potential to do damage, whereas if more elevation is available then heavier rods are more damaging with a 5.65kg rod having the highest energy when falling from 50m. In such cases the rod will have travelled horizontally about 28m and will have impact energy levels just over 400J.

4 CONCLUSIONS

Approximate solutions to the windborne debris trajectory equations have been derived. These have been used to investigate the damage potential of various rods, ranging from poly-styrene to steel. Interestingly it is found that rods with densities similar to timber are likely to be the most damaging when falling from a range of heights up to 50m in a 20 m/s wind.

5 REFERENCES

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