15th Australasian Wind Engineering Society Workshop
Sydney, Australia
23-24 February 2012

Drag coefficients for roughened circular cylinders in super-critical flow

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Introduction

Although information on flow and force coefficients for two-dimensional circular cylinders at sub-critical Reynolds Numbers (below about 1×10^5 for cylinders with nominally 'smooth' surfaces) is widely available, there is less data in the critical range up to the minimum drag coefficient, and very few studies in the super-critical range up to Re_b equal to 10^6 and beyond. However, the flow around most structures with circular cross sections such as lighting poles, chimneys and observation towers at wind speeds for structural design at ultimate limit states, falls into the critical or super-critical ranges, where the information is most sparse. For high wind speeds, this is also the case for members of lattice towers, such as flare towers of petrochemical plants and LNG facilities. The limited experimental data that are available often suffer from issues such as uncorrected blockage effects, and lack of two-dimensionality due to low aspect ratios – these often being bi-products of attempts to achieve high Reynolds numbers in conventional wind tunnels.

This paper will review past studies of drag coefficients for two-dimensional circular cylinders, in smooth uniform flow, in the super-critical range, and will also discuss the use of roughness to simulate super-critical flow at lower Reynolds Numbers. Some new experimental data for roughened cylinders are also presented.

Some comments on the application to circular cylindrical members and structures in turbulent atmospheric flow are made.

Previous work

One of the first serious attempts to measure drag of smooth circular cylinders in the super-critical range was that of Roshko (1960) who was able to achieve a Reynolds Number of 9×10^6 in a high-pressure wind tunnel. He found the drag coefficient increased in the range $2 \times 10^6 < Re_b < 3.5 \times 10^6$. For $3.5 \times 10^6 < Re_b < 9 \times 10^6$, Roshko found the drag coefficient to be relatively constant at a value of about 0.7.

Schewe (1983) also carried out measurements in a pressurized wind tunnel over a wide range of Reynolds Numbers between 2.3 9×10^4 and 7.1×10^6. Beyond 5×10^6, he found a constant value of drag coefficient of 0.52; he noted the higher value measured by Roshko, and attributed it to higher surface roughness in the latter case, although Schewe did not make any correction for blockage effects.

Surface roughness has a significant effect on the flow around circular cylinders – particularly in the super-critical range – as noted by Fage and Warsap (1930). Fage and Warsap measured the drag on cylinders of two different diameters with six different types of sandpaper providing varying roughness heights. In these classic experiments the reduction of the critical Reynolds Number with increasing roughness height was noted.

Measurements of drag coefficients on cylinders with deliberately roughened surfaces, to high Reynolds Numbers, were also described by Achenbach (1971), Szechenyi (1975), Guven *et al.* (1980), Nakamura and Tomonari (1982), Shih *et al.* (1993) and Adachi (1995). Some of these data have been re-plotted for this paper, and are discussed in later sections.

Minimum drag coefficient

The Reynolds Number based on diameter (Re_b) defining the lower limit of the super-critical flow range, can be taken to be that at which the drag coefficient reaches a minimum value, Re_{minCd}. The latter value, and the minimum drag coefficient C_{dmin}, are both functions of the ratio of average surface roughness height, k, to the diameter, b. As the ratio k/b increases, Re_{minCd}.decreases, and C_{dmin} increases.

Figure 1 shows Re_{minCd} plotted against k/b for the results of Achenbach (1971), Fage and Warsap (1930), and Guven et al. (1980). The values are well fitted with a straight line on log-log axes, indicating a power relationship as follows:

$$Re_{minCd} \cong 4210\ (k/b)^{-0.555} \qquad (1)$$

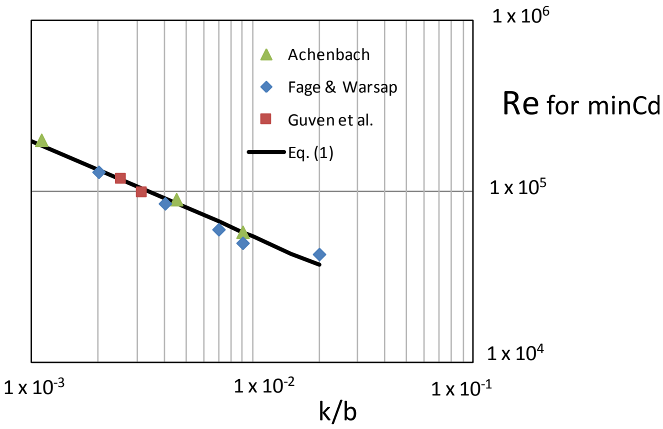

Figure 1. Reynolds Number (based on diameter) for minimum drag coefficient, as a function of the ratio of average roughness height to diameter.

For (k/b) less than 10^{-3}, the value of Re_{minCd} is nearly constant – i.e. it is independent of (k/b) for relatively smooth cylinders (Adachi, 1995).

Roughness Reynolds Number

Szechenyi (1975) replotted the results of Fage and Warsap, together with some new data, and showed that the drag coefficients, in the super-critical range (i.e. for $Re_b > Re_{minCd}$), effectively collapsed when they were plotted against a *roughness Reynolds Number*, Re_k, defined as (Vk/ν), where V is the wind speed, k is the average roughness height, and ν is the kinematic viscosity. Data from a nominally smooth cylinder were included by Szechenyi by assuming a relative roughness (k/b) of 3.5×10^{-5}.

Figure 2 shows the data of Fage and Warsap, and of Szechenyi, when plotted in this way. Drag coefficients by Achenbach (1971) and Guven *et al.* (1980) are also re-plotted against roughness Reynolds Number in this figure. With the exception of some of the data of Achenbach, the collapse within each data set is good. However, there is some inconsistency in data from different authors. This most likely can be attributed to errors resulting from low aspect ratios, high blockage, and significant amounts of turbulence in the flow. Also, most of the measurements shown are based on pressure measurements with finite pressure tap spacings – requiring integration to obtain drag values. However, the data shown in Figure 2 confirms the effectiveness of the roughness Reynolds Number in collapsing drag coefficients in the super-critical range. Additional measurements were carried out at Monash University, with some care taken to avoid the potential experimental errors described above; these are discussed in the following section.

Drag measurements at Monash University

Measurements of drag on roughened circular cylinders were carried out in an extension to the high-speed jet erected in the plenum of the large (1.4 MW) wind tunnel at Monash University.

Cylinders of 38mm or 76mm diameter were connected to an underfloor force balance system (Figure 3). Fixed 'dummy' cylinders were mounted from the top of the test section, with a small gap between the 'active' and 'dummy' cylinders (Figure 4). This gave effective span/diameter aspect ratios of 25 or 50.

The cylinders were roughened with a mixture of sand particles (two sizes) and paint (Figure 5), giving average roughness heights of about 140 microns (0.14mm) and 350 microns (0.35mm). The drag measurements were carried out in wind speeds between about 5 m/s and 50 m/s, giving Reynolds Numbers (based on diameter) between 1.4×10^4 and 2.5×10^5. The flow was effectively smooth (turbulence intensity of 1-2%).

Figure 3. A circular cylinder set up for measurement of drag coefficient in the 1.5 MW Monash University wind tunnel

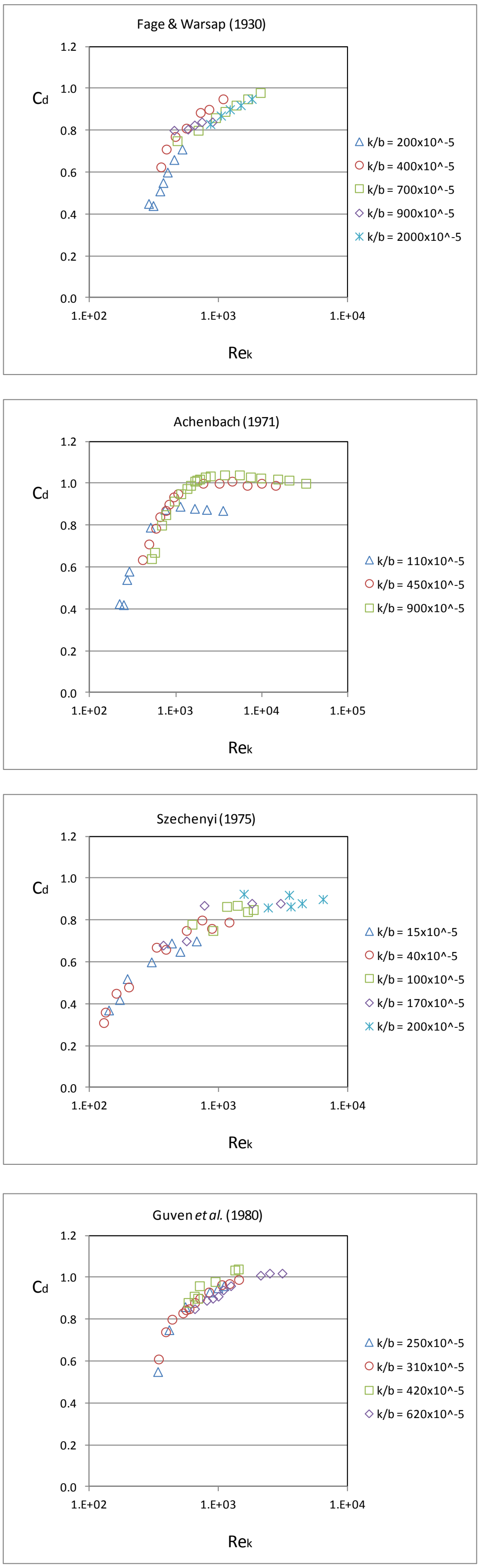

Figure 2. Drag coefficients in super-critical flow plotted against roughness Reynolds Number, Re_k.

(a) Fage andWarsap (1930); (b) Achenbach (1971); (c) Szechenyi (1975); (d) Guven *et al.* (1980).

Figure 4. Close-up view of the gap between the upper (dummy) and lower (active) cylinders.

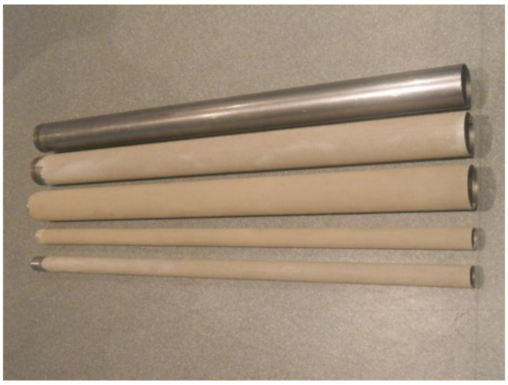

Figure 5. Various smooth and roughened cylinders tested.

Figure 6 shows the drag coefficients for three roughened cylinders plotted against the Reynolds Number based on diameter (Re_b). The critical range, in which the drag coefficient falls sharply over a narrow range of Re_b, the minimum drag coefficient, and the super-critical range with rising drag coefficients, are easily identified in these graphs.

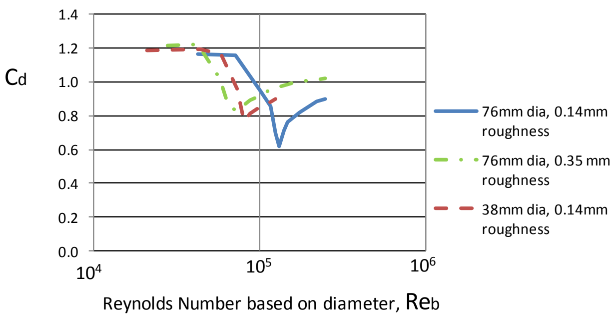

Figure 6. Drag coefficient for roughened cylinders versus Reynolds Number, Re_b (Monash University tests)

Re-plotting the data from the super-critical range in Figure 6 against roughness Reynolds Number, Re_k, gives the plot in Figure 7. This shows that the drag coefficients do indeed collapse well when plotted in this way, generally supporting the approach suggested by Szechenyi (1975).

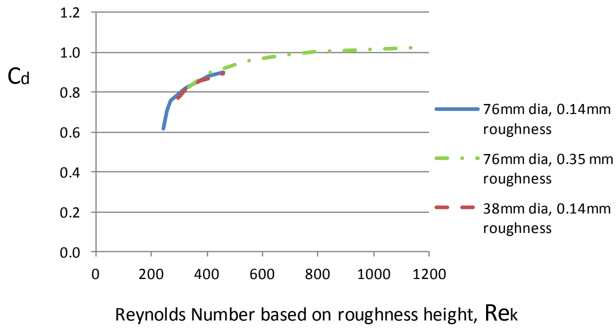

Figure 7. Drag coefficient for roughened cylinders versus roughness Reynolds Number, Re_k (super-critical flow - Monash University tests)

Discussion and limitations

Szechenyi (1975) in introducing the roughness Reynolds Number concept, proposed limitations on the super-critical flow range. These were:

$Re_k > 200$, and $(k/b) < 2.2 \times 10^{-3}$

Although the first condition applies to all the measurements shown in Figures 2 and 6, many measurements do not satisfy the second condition, with roughness/diameter ratios up to 2×10^{-2} in the Fage and Warsap (1930) tests. However, generally the data in Fig. 2 does seem to collapse well for a range of k/b from 1.4×10^{-4} to 2.2×10^{-2}.

Eq. (1) would suggest that for super-critical flow, and (k/b) *greater* than 10^{-3}, Re_k should be greater than $4210\ (k/b)^{0.455}$. For (k/b) *less* than 10^{-3}, Re_k should be greater than $1.95{\times}10^5\ (k/b)$. These two relationships both give Re_k greater than 195 for k/b equal to 10^{-3}. Thus, Szechenyi's lower limit of 200 on Re_k for all (k/b), seems over-simplified.

At very high values of both Re_b and (k/b), Guven *et al.* (1980) have suggested that the drag coefficient becomes independent of both parameters and hence of Re_k. This apparently is the 'transcritical' flow range as proposed by Roshko (1960) for smooth cylinders. The value of Re_k above which the drag coefficient becomes constant appears to be around 2000, based on the measurements shown in Figure 2. The value of C_d for this range is about 1.0 to 1.1. Although Szechenyi himself found a lower value of around 0.9 (see Figure 2c), his measurements were probably affected by low aspect ratios, and uncorrected blockage errors (in a perforated-wall wind tunnel).

Turbulence effects, and applications to model testing

Of course, wind flows around objects and structures with circular cross-sections in the atmospheric boundary layer are turbulent, and it is well known that turbulence, at high turbulence intensities, have a significant effect on the drag coefficients of circular cylinders (e.g. Cheung and Melbourne, 1983). However the length scales of turbulence in the atmosphere are very large compared to the diameters of typical members and structures. For example, the ratio of turbulence integral scale to diameter for a tubular structural member with a diameter of 100 mm is about 1000, and the ratio is about 100 for a chimney with a diameter of 1 metre. For those ratios, the effect of turbulence on the flow around the cylinders can probably be considered as quasi-steady, and similar to smooth flow. Of course the situation is different for large structures with a circular cross-section, such as a cooling tower with a diameter of 30 metres.

However, further investigation is required as to the effect of surface roughness combined with high turbulence intensities on the drag of circular cylinders. If the roughness Reynolds

Number scaling applies in that situation, it opens up the idea of simulating super-critical flow on large structures in the atmosphere by the use of high roughness on smaller models at lower wind speeds in wind tunnels. That idea was suggested, in fact, as early as 1968, for simulation of wind effects on cooling towers with small-scale wind-tunnel models by Armitt (1968). It has not been widely adopted in the last forty years, but it deserves renewed attention. Of course for this method to work it is necessary that the flow in *both* full-scale and model scale be super-critical. This can be checked using the criteria discussed in the previous section.

Application to design codes and standards

The Eurocode (B.S.I., 2005) and ESDU 80025 (ESDU International, 1986) both provide comprehensive information on drag coefficients in super-critical flow, including the effects of surface roughness. However, neither of these documents have adopted the roughness Reynolds Number approach of Szechenyi, although this would clearly have simplified the presentation of the data.

The Eurocode (B.S.I., 2005), in Figure 7.28, gives varying values of drag coefficients for circular cylinders in super-critical flow, in a graphical form depending on the Reynolds Number Re_b, with several lines for varying ratios of roughness height, k, to diameter (k/b), together with the corresponding mathematical equation. The function given in the Eurocode, for super-critical flow, has been re-plotted against Re_k in Figure 8. Except for the case of $k/b = 10^{-2}$ (an uncommon situation for full-scale structures) the collapse of the Eurocode data is very good when re-plotted in this way. An enveloping function for the Eurocode data is as follows:

$$C_d = 0.25 + 0.275 (\log_{10} Re_k) - 0.025 (\log_{10} Re_k)^2 \qquad (2)$$

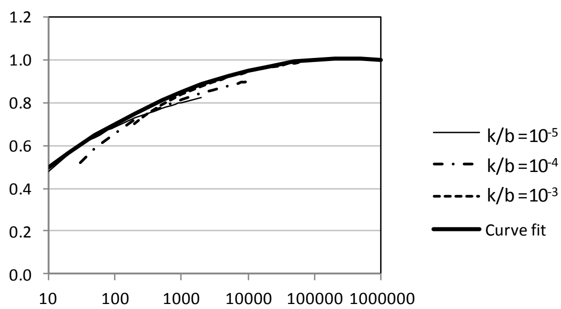

Figure 8. Re-plot of drag coefficients from Eurocode (Fig. 7.28) plotted against roughness Reynolds Number, Re_k, with enveloping curve fit

Equation (2) is simpler than the function provided in the Eurocode, and could be used as a replacement in a code format. It is generally conservative compared with most of the experimental data, particularly for Re_k less than about 300 (see Figures 2 and 7), but that may not be an unreasonable property for a formula for general use in a code or standard.

Conclusions

This paper has reviewed past studies of drag coefficients for two-dimensional circular cylinders in the super-critical range, with a range of diameters and roughness heights, and, following the approach of Szechenyi (1975), shown that the data consistently 'collapse' when plotted against a 'roughness Reynolds Number', composed of the product of the Reynolds Number based on diameter, Re_b, and the ratio of roughness height to diameter (k/b). Some new experimental data have been presented which confirms the roughness Reynolds Number approach.

The paper has also discussed the use of roughness to simulate super-critical flow at lower Reynolds Numbers on wind-tunnel models, and a possible formula for use in codes and standards, based on graphs given in the Eurocode.

Acknowledgments

The assistance of Monash University technical staff, particularly Don McMaster, in manufacturing the models and with the wind-tunnel testing, is acknowledged by the authors.

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