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Prevention study on rain-wind-induced vibration of inclined cables

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Abstract

Rain-wind-induced vibration of an inclined cable is a large amplitude vibration with potential fatigue damage on structures. This paper addresses prevention design for this kind of vibration with additionally fixed dampers. Both the influences of axial and out-of-plane vibration of a cable being neglected, the in-plane vibration modes are used to simulate its dynamic curve. Considering the interactions between cable and water rivulet and the influence of water rivulet position on aerodynamic forces, a two-degree-of-freedom nonlinear model of the coupling system is developed. Then Hurwitz discriminant is applied to evaluate the kinematic stability of the system. When the damper of the system is negative, self-excited vibration of the cable occurs. That is the mechanism of rain-wind-induced vibration. The requirement for additional dampers is studied based on the stability assessment criterion. Finally, an example is given to show that the proposed analytical model and the mitigation design procedure are reasonable and effective.

Introduction

Rain-wind-induced vibration of an inclined cable is a severe vibration with large amplitude, which might cause potential fatigue damage in short periods and should be mitigated in engineering. Such dynamic behavior is a solid-liquid-wind interaction problem with complicated mechanism. The forming and the moving of water rivulet on the surface of a cable in wind and rain circumstance changes cable section and aerodynamic forces; the latter, on the other hand, affects the vibration of the cable and the water rivulet. The interactions among the cable, the water rivulet and aerodynamic forces induce self-excited vibration of the system, namely rain-wind-induced vibration. Because of so many infecting factors and the nonlinear characteristics, the mechanism of such vibration is complex and difficult to analyze.

Hikami and Shiaishi (1988) firstly observed rain-wind-induced vibrations on Meikonishi Bridge in Nagoya, Japan, where the amplitudes of inclined cables were observed up to 55cm under wind of velocity 14m/s. During the vibration, a water rivulet was observed to appear on the lower surface of the cable, oscillating in circumferential direction with the same frequency of the cable. Further wind tunnel experimental research showed that the cable oscillations were mostly of single mode from the 1st to the 4th, in the vertical plane and that the formation position of water rivulets depended on mean wind velocity. Based on further wind tunnel test results and field measurement results, Matsumoto et al (1992, 2003) concluded that the formation of upper water rivulet and the axial flowing might be the inducement of rain-wind-induced vibrations. Bosdoginni and Oliver (1996) compared the tunnel test results between fixed water rivulet model and moving water rivulet model and indicated that the position, not the moving, of upper water rivulet was the primary cause of the vibration.

Regarding theoretical analysis, Yamagushi (1990) proposed a two-degree-of-freedom galloping model, considering the cable as a horizontal rigid cylinder. After Xu & Wang (2003) and Wilde & Witkowski (2003) described the movement of rivulet as simple harmonic circumferential oscillating at the frequency of the cable, the plane model was further studied, and the analytical results were compared with those from wind tunnel tests. It turned out that such analytical models could capture main dynamic features of inclined cylinders with either moving rivulet or artificial fixed rivulet.

As a further study, this paper addresses a theoretical study of dynamics of an integral cable controlled through additional dampers. The in-plane mode is used to simulate the dynamic curve of the cable, and the oscillation of rivulet is explored. After the interactions among cable, damper, rivulet and aerodynamic forces are studied, the governing equation for the coupling system is derived. Hurwitz discriminant is applied to evaluate the kinematic stability of the model. By means of the harmonic balance method, algebra equations with the steady amplitudes are developed. Finally, an example is given to verify the effectiveness of the proposed mitigation design procedure.

Analytical Model

Theoretical assumption

The primary assumptions in this paper are as follows:

(1) Mass, shape and size of rivulet are negligible compared with those of the cable (Irvine, 1981).

(2) Initial position of rivulet is a function of mean wind velocity (Hikami & Shiaishi 1988).

(3) The steady wind force coefficients are obtained from Gu (2002).

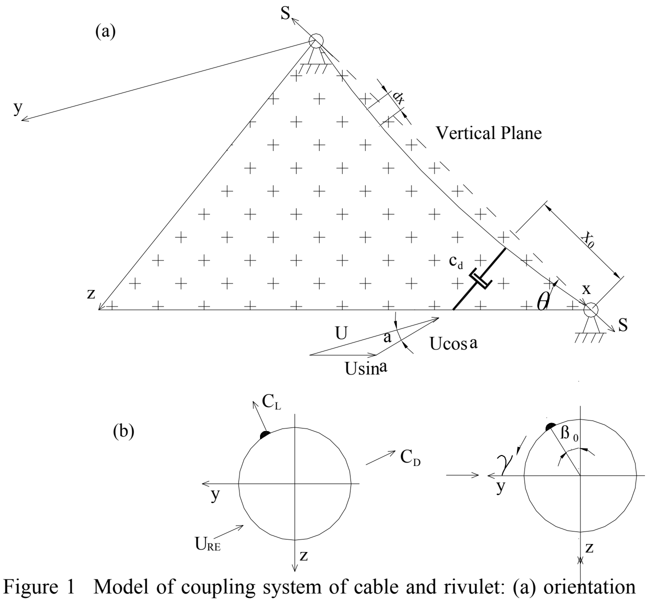

Figure 1 Model of coupling system of cable and rivulet: (a) orientation of inclined cable; (b) relative wind velocity.

Neglecting the influence of rivulet oscillation on relative wind velocity, the aerodynamic force on the unit length of the cable is:

$$F_z = -\rho D U_{RE}^2 (C_D \cdot \sin\phi^* + C_L \cdot \cos\phi^*)/2 \tag{1}$$

where ρ is the fluid density, D is the diameter of the cable, $C_L(\phi_e)$ 、 $C_D(\phi_e)$ are lift and drag coefficient, respectively, and

$$U_{RE} = \sqrt{(U\sin\alpha\sin\theta + \dot{w})^2 + (U\cos\alpha)^2} \tag{2}$$

$$\phi^* = \arctan(\frac{U\sin\alpha\sin\theta + \dot{w}}{U\cos\alpha}) \tag{3}$$

$$\phi_e = \phi^* - \gamma - \beta_0 \tag{4}$$

where $\dot{w} = dw/dt$ and t is the time.

Differential equations of motion

The differential equation for in-plane motion of a cable is written as follows:

$$(S+h)\frac{\partial^2 w}{\partial x^2} + h\frac{d^2 z_q}{dx^2} - \left[c_{w0} + c_d \delta(x - X_0)\right]\frac{\partial w}{\partial t} + F_z = M\frac{\partial^2 w}{\partial t^2} \tag{5}$$

where h(t) is the dynamic increment of cable tension, which can be calculated as [13]:

$$h = \frac{EA}{l}\int_0^l \left[(dz_1/dx)\cdot(\partial w/\partial x) + (\partial w/\partial x)^2/2\right]dx \tag{6}$$

Here $z_1(x)$ is the initial curve function of the cable, $w(x,t)$ is the vertical dynamic displacement, EA is the section stiffness, S_0 and l are the initial tension and length, M is the mass per unit length, c_{w0} is the damping coefficient for cable, $\delta(x)$ is the unit impulse or Dirac delta function, defined as

$$\delta(x) = \begin{cases} \infty & x = 0 \\ 0 & x \neq 0 \end{cases} \quad and \quad \int_{-\infty}^{+\infty}\delta(x)dx = 1 \cdot$$

The kinetic energy of water rivulet per unit length is,

$$dT = m[(\dot{w} + D\dot{\gamma}\sin\beta/2)^2 + (D\dot{\gamma}\cos\beta/2)^2]/2dx \tag{7}$$

where m is the mass per unit length of the rivulet, β is the instantaneous position of the rivulet: $\beta(x,t) = \beta_0(x) + \gamma(x,t)$.

The governing equation for motion of rivulet can be obtained from Lagrange equation,

$$\frac{d}{dt}(\frac{\partial dT}{\partial \dot{\gamma}}) - \frac{\partial dT}{\partial \gamma} = -c_\gamma|\dot{\gamma}|\dot{\gamma} \tag{8}$$

namely

$$\ddot{\gamma} + c_\gamma|\dot{\gamma}|\dot{\gamma} = -\frac{2}{D}\ddot{w}\sin\beta \tag{9}$$

where, c_γ is the damping coefficient of rivulet.

Governing equations for coupling system of cable and rivulet

As the cable oscillations are mostly of single mode in the vertical plane, the dynamic curve of the cable can be simulated through the mode function as,

$$w(x,t) = q_w(t)\phi_w(x) \tag{10}$$

Substituting equation (10) into equation (9) and retaining only the linear terms of γ and q_w the dynamic response of rivulet is obtained as

$$\gamma(x,t) = q_\gamma(t)\phi_\gamma(x) \tag{11}$$

$$\phi_\gamma(x) = \sin\beta_0(x)\phi_w(x) \tag{11a}$$

where $q_w(t), q_\gamma(t)$ are amplitudes of cable and rivulet, respectively ; ϕ_w is the in-plane mode function of the cable.

Substitute equation(10),(11) into equation(5),(6) and (9), one gets,

$$\ddot{q}_w + c_w\dot{q}_w + \omega_w^2 q_w = f_w + g_w \tag{12}$$

$$\ddot{q}_\gamma + c_\gamma^* A_\gamma \dot{q}_\gamma = f_\gamma \tag{13}$$

$$f_w = \int_0^l F_z\phi_w dx/(M\int_0^l \phi_w^2 dx) \tag{14}$$

$$g_w = a_1 q_w^2 + a_2 q_w^3 \tag{15}$$

$$a_1 = \frac{EA}{Ml\int_0^l \phi_w^2 dx}\int_0^l \phi_w''\phi_w dx\int_0^l z'\phi_w' dx \tag{16a}$$

$$a_2 = \frac{EA}{2Ml\int_0^l \phi_w^2 dx}\int_0^l \phi_w''\phi_w dx\int_0^l \phi_w'^2 dx \tag{16b}$$

$$f_\gamma = -2(\ddot{q}_w\int_0^l \sin\beta\phi_\gamma\phi_w dx)/(D\int_0^l \phi_\gamma^2 dx) \tag{17}$$

$$c_w = \left[c_{w0} + c_d\phi_w^2(X_0)\right]/(M\int_0^l \phi_w^2 dx) \tag{18}$$

$$c_\gamma^* = 8c_\gamma\omega_w\int_0^l \varphi_\gamma^3 dx/(3\pi\int_0^l \varphi_\gamma^2 dx) \tag{19}$$

where ω_w is the circular frequency of the cable, A_γ is the angle amplitude of rivulet; symbols "." and ". ." denote the first and the second order derivation with respect to time, while symbols' and'' represent the first and the second order derivatives with respect to x.

Expand equation (14), (17) with Taylor Series, neglect the 4th (and higher) power of the response, one gets,

$$f_w = b_0 + b_1\dot{q}_w + b_2\dot{q}_w^2 + b_3\dot{q}_w^3 + b_4\dot{q}_w q_\gamma^2 + b_5\dot{q}_w^2 q_\gamma + b_6\dot{q}_w q_\gamma + b_7 q_\gamma + b_8 q_\gamma^2 + b_9 q_\gamma^3 \tag{20}$$

$$f_\gamma = -m^* \ddot{q}_w (1 + q_\gamma d_1 - q_\gamma^2 d_2 / 2) \tag{21}$$

Where $d_1 = \int_0^l \cos\beta_0 \phi_w \phi_\gamma^3 dx / \int_0^l \phi_\gamma^2 dx$;

$d_2 = \int_0^l \sin\beta_0 \phi_\gamma^3 dx / \int_0^l \phi_\gamma^2 dx \, , m^* = 2/D$.

Equation (12), (13) can also be expressed in matrix forms as,

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{P} \tag{22}$$

where $\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $\mathbf{C} = \begin{bmatrix} c_w & 0 \\ 0 & c_\gamma^* A_\gamma \end{bmatrix}$;

$\mathbf{K} = \begin{bmatrix} \omega_w^2 & 0 \\ 0 & 0 \end{bmatrix}$; $\mathbf{q} = \begin{bmatrix} q_w \\ q_\gamma \end{bmatrix}$; $\mathbf{P} = \begin{bmatrix} f_w + g_w \\ f_\gamma \end{bmatrix}$

Equation (22) is the governing equation of two-degree-of-freedom for rain-wind-induced vibration of inclined cables.

Judgment for the stability of the governing equation

The stability analysis for $\mathbf{q} = \mathbf{0}$ can be performed by linearizing equation (22), which is rewritten as

$$\mathbf{M}^*\ddot{\mathbf{q}} + \mathbf{C}^*\dot{\mathbf{q}} + \mathbf{K}^*\mathbf{q} = \mathbf{0} \tag{23}$$

where $\mathbf{M}^* = \begin{bmatrix} 1 & \\ m^* & 1 \end{bmatrix}$; $\mathbf{C}^* = \begin{bmatrix} c_{11} & 0 \\ 0 & 0 \end{bmatrix}$;

$\mathbf{K}^* = \begin{bmatrix} \omega_w^2 & k_\gamma \\ 0 & 0 \end{bmatrix}$; $c_{11} = c_w - b_1$.

The characteristic polynomial of equation (23) is

$$\begin{aligned} G(\lambda) &= \det(\mathbf{M}^* \cdot \lambda^2 + \mathbf{C}^* \cdot \lambda + \mathbf{K}^*) \\ &= \lambda^2 (\lambda^2 + e_1 \lambda + e_2) \end{aligned} \tag{24}$$

where $e_1 = c_{11}$, $e_2 = \omega_w^2 - m_\gamma^* k_\gamma$.

Equation (23) remains stable, when all the eigenvalues of equation (24) has negative real part. According to Hurwitz criterion, the condition requires that the inequalities

$$g_i > 0 \qquad i = 1,2 \tag{25}$$

be satisfied, where $g_1 = e_1$, $g_2 = e_1 e_2$. Since e_2 is positive for usual speed velocity, the stability of equation (23) only depends on the sign of e_1 , or c_{11}. When damper of the system c_{11} is positive, the system remains stable; otherwise rain-wind-induced vibration occurs.

Calculations for cable amplitude

As the nonlinear terms of response appear in the right side of equation (22), the amplitude of cables will not increase unlimitedly. There exists steady vibration with constant amplitude for the system when rain-wind-induced vibration occurs. The harmonic balance method is employed in the paper.

The dynamic response of steady vibration of the cable can be expressed as

$$q_w = A_w \cos\omega t \tag{26a}$$

$$\dot{q}_w = -\omega A_w \sin\omega t \tag{26b}$$

$$\ddot{q}_w = -\omega^2 A_w \cos\omega t \tag{26c}$$

Considering phase difference, the dynamic response of rivulet can be expressed as

$$q_\gamma = A_{\gamma 1} \cos\omega t + A_{\gamma 2} \sin\omega t \tag{27a}$$

$$\dot{q}_\gamma = -\omega A_{\gamma 1} \sin\omega t + \omega A_{\gamma 2} \cos\omega t \tag{27b}$$

$$q_\gamma = -\omega^2 (A_{\gamma 1} \cos\omega t + A_{\gamma 2} \sin\omega t) \tag{27c}$$

Substituting (26) and (27) into (22), and considering that coefficients of $\cos\omega t, \sin\omega t$ equal to those on the other side of the equation, one gets

$$\begin{aligned} A_w(-\omega^2 + \omega_w^2) &= b_7 A_{\gamma 1} + \frac{1}{4} b_5 \omega^2 A_w^2 A_{\gamma 1} \\ &- \frac{1}{2} b_4 \omega A_w A_{\gamma 1} A_{\gamma 2} + \frac{3}{4}(a_2 A_w^3 + b_9 A_{\gamma 1}^3 + b_9 A_{\gamma 1} A_{\gamma 2}^2) \end{aligned} \tag{28a}$$

$$\begin{aligned} -\omega c_{11} A_w &= b_7 A_{\gamma 2} - \frac{3}{4}(b_3 \omega^3 A_w^3 - b_5 \omega^2 A_w^2 A_{\gamma 2}) \\ &- \frac{1}{4} b_4 \omega A_w (A_{\gamma 1}^2 + 3A_{\gamma 2}^2) + \frac{3}{4} b_9 A_{\gamma 2} (A_{\gamma 1}^2 + A_{\gamma 2}^2) \end{aligned} \tag{28b}$$

$$\begin{aligned} &-\omega A_{\gamma 1} + c_\gamma^* \sqrt{A_{\gamma 1}^2 + A_{\gamma 2}^2} A_{\gamma 2} \\ &= m^* \omega A_w (1 + \frac{3}{8} d_2 A_{\gamma 1}^2 + \frac{1}{8} d_2 A_{\gamma 2}^2) \end{aligned} \tag{28c}$$

$$-\omega A_{\gamma 2} + c_\gamma^* \sqrt{A_{\gamma 1}^2 + A_{\gamma 2}^2} A_{\gamma 1} = \frac{1}{4} d_2 m^* \omega A_w A_{\gamma 1} A_{\gamma 2} \tag{28d}$$

Thus the nonlinear governing equation (22) is changed into an algebra equation group (28), containing unknowns $\omega, A_w, A_{\gamma 1}, A_{\gamma 2}$.

Numerical example

The cable under consideration has following properties: length l=154.3m , diameter D=0.190m , initial tension T= $7.92 \times 10^6 N$, damping ratio for cable only ξ=0.2%, mass per unit length m=149kg/m, inclination angle θ =40.3°, yawing angle α =72°. The first three frequencies of the cable are 0.74Hz, 1.48Hz, 2.22Hz, respectively. The initial position of rivulets and aerodynamic coefficients are from (Hikami and Shiraishi, 1988; Gu et al., 2003).

Firstly, we do not include any additional dampers, e.g. c_d=0. The dynamic responses of the cable for the first three frequencies are studied. Fig. 2 gives the variations of the stable parameter g1 vs. wind velocity. The result shows that the cable is unstable with wind velocity 7.5~14m/s. In fig. 3, the calculated amplitudes of the cable are compared with the field measuring ones (Takano et al, 1997). Both of calculated results and field measuring results show that the amplitude of a lower rank mode is greater than that of a higher rank mode. Fig. 6 gives the corresponding maximum oscillating angle of rivulet.

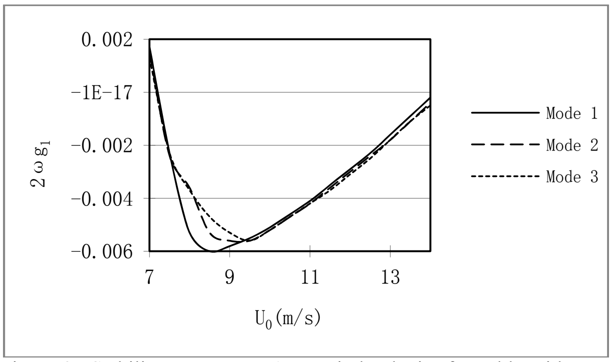

Figure 2 Stability parameter g1 vs. wind velocity for cable without any damper control

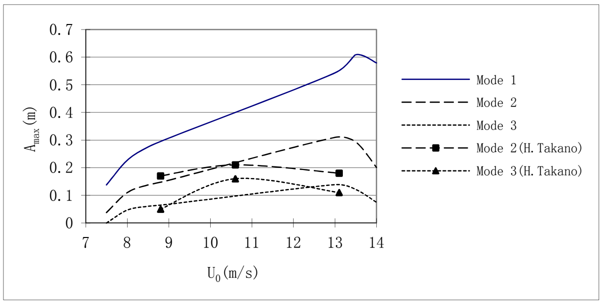

Figure 3. The maximum amplitude-in-plane of cable without damper control vs. wind velocity

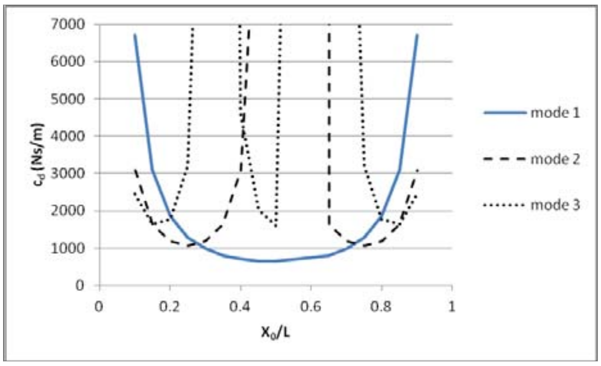

Figure 4. Minimum coefficient for additional damper to prevent rein-wind-induced vibration

To prevent rain-wind induced vibration of inclined cables, the inequality (25) should be satisfied for all wind velocity and all yawing angle. The smallest c_d required for the non rain-wind vibration in the example is shown in fig. 4, indicating that optimum additional damper position differs for different vibration modes. For this case, an additional damper with coefficient 3110 N-s/m installed between 0.15-0.25L or 0.75-0.85L will be enough to prevent rain-wind induced vibration for the first three modes.

Conclusion

This paper addresses the prevention design of rain-wind-induced vibration of inclined cables. The oscillation of rivulet is explored through considering the in-plane modes of inclined cables. After the interactions among cable, damper, rivulet and aerodynamic forces are studied, the governing equation for the coupling system is derived. Hurwitz discriminant is applied to evaluate the kinematic stability of the model. By means of the harmonic balance method, the steady amplitudes are calculated. The prevention of the severe vibration requires positive values of the stability parameters, based on which, the coefficient of the additional damper is optimized in terms of the damper position. An example is given to verify the effectiveness of the proposed analytical model and mitigation design procedure. Numerical results show that the optimum damper position is between 0.15-0.25L or 0.75-0.85L of an inclined cable to prevent the first three modes of rain-wind induced vibrations.

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