**Design Criteria For Reinforced Concrete Chimneys Under Vortex Excitation**

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## 1. INTRODUCTION

In the most recent edition, the reinforced concrete chimney code [ACI-307-1988] changed from criteria based on working stresses to criteria based upon ultimate capacity and factored loads. Along with the changes came an inconsistency. The drag wind loading and the earthquake loading can be logically treated by load factor and ultimate capacity since this is essentially a limit state approach. The load factor ensures that the load used has an acceptable low probability of occurrence and the factored load can be associated with an event having a probability of occurrence that can be readily estimated. In assessing the behaviour under these extreme conditions it is also logical to presume a damping capacity consistent with extensive cracking and yielding ie. approaching 5% of critical. In the case of vortex-induced loads the maximum effects are often developed at wind speeds much less than say the 50-year speed and the above approach is flawed. In such circumstances it is expected that the structure will remain in good condition and a limiting stress criterion is more reasonable. It is also logical to assume a damping level consistent with this behaviour ie. about 1% of critical.

This paper sets out to define acceptable stress levels for vortex-induced loads. These stresses are defined by ensuring that they result in an acceptable fatigue life for the chimney. Because the aim is to define stresses suitable for inclusion in a code and essentially independent of the particular structure it is necessary to make a number of simplifying assumptions which, as far as is reasonable, lead to conservative estimates of the acceptable stress levels.

## 2. SIMPLIFYING ASSUMPTIONS

### 2.1 Wind-Climate

The essential assumption in regard to the wind climate is that the probability of exceedance of a mean speed $\bar{V}$ is adequately approximated by the Weibull form;

$$P(>V) = \exp\{-(V/C)^{\gamma}\}$$

and that the mean speed with a return period of 50 years corresponds to a value of $P(>V)$ of $2.3 \times 10^{-6}$. Fatigue computations assume a value of $\gamma$ of 2.0 but the sensitivity to $\gamma$ is examined. The above assumption tends to be conservative since in most climates some of the extremes tend to be associated with storm types that do not dominate the general climate and hence values of $C$ derived from the 50-year wind speed will normally be high. In computing fatigue damage it is assumed that the climate is uni-directional. For an axi-symmetric structure a uni-directional climate and a uniform climate are the bounds. It is demonstrated that in terms of the allowable stress required to achieve a specified fatigue life the two bounds differs by about 10% to 15% with the uni-directional assumption being conservative.

### 2.2 Dependence of Response (Moment) on Wind Speed

The ratio of the root-mean-square response at the critical mean speed, $V_c = \frac{1}{S} f_o D$, to that at any other mean speed $V$ is given by the relationship;

$$\frac{\sigma}{\sigma_c} = K^{3/2} \exp\left\{-(1-K^{-1})^2/2B^2\right\}$$

where $K = V/V_c$ and $B$ is a bandwidth parameter with a value, typically, between 0.2 and 0.4 and commonly near 0.3. Due to motion-induced loads the foregoing relationship tends to slightly overestimate values of $\sigma/\sigma_c$ away from $K = 1$, this error is conservative in regard to the fatigue computations.

### 2.3 Probability Distribution of Vortex-Induced Moments

The distribution of the envelope "a" of the moments is assumed to be of the Rayleigh form ie;

$$P(>a) = e^{-a^2/2\sigma^2}$$

where $P(>a)$ is the probability of the envelope exceeding $a$ and $\sigma$ is the standard deviation.

### 2.4 Fatigue Damage

The fatigue life is computed in accordance with Miner's Law with the damage per cycle ($D=N_f^{-1}$, where $N_f$ is the cycles to failure) being determined from the relationships;

#### 2.4.1 Reinforcing steel [grade 60]; (simplified from [1])

$\log_{10} N_f = 6.80 - 0.0059 f_r$ where $f_r = f_{max} - 2/3\, f_{min}$ (MPa), (tension positive)

and $D = 0$ for $f_r < 163$ MPa.

#### 2.4.2 Concrete; (simplified from [2])

$\log_{10} N_f = 15(1 - f_m / kf_c')$ where $f_m$ = maximum compressive stress

$f_c'$ = compressive strength $k$ = a reduction factor of say, 0.85.

### 2.5 Moment/Stress Relationship

The moment/stress relationship for reinforced concrete chimneys is non-linear and has a form as depicted in Fig. 1 (steel) and Fig. 2 (concrete) [3]. The initial dead load compression and the assumption of zero tensile strength for the concrete leads to stress moment curves which are concave away from the initial stress line. While this non-linearity presents no difficulties for a specific fatigue analysis it does present a problem in the present case where the intention is to develop more general results suited to inclusion in a code and not dependent upon the actual section properties and preload.

The approach taken here was to replace the true non-linear relationship by a linear function which matched the true at a specific point. The assumed and true relationships are matched at the bending moment corresponding to excitation with $V=V_c$ and a value of $a/\sigma$ of 4.0. This ensures that there will be very few cycles to the right of the matching point and hence that the stress range will almost always be overestimated and the estimate of fatigue damage conservative. Matching as described above is not unconditionally conservative (eg. if the endurance limit is to the right of the matching point) but any unconservative errors will undoubtedly be small.

## 3. COMPUTATION OF FATIGUE DAMAGE

The expected damage per cycle ($\bar{D}$) is given by

$$\bar{D} = \alpha \int_{V=0}^{\infty} p(v) \int_{a=0}^{\infty} p(a/v) D(a)\, da\, dV \quad ; \text{where}$$

$$p(v) = \frac{24.5V}{V_{50}^2} \exp\left\{-\left(\frac{3.5V}{V_{50}}\right)^2\right\} \qquad \text{for } \gamma = 2$$

$$p(a/v) = \frac{16a}{\hat{a}_v^2} \cdot \exp\left\{-\frac{8a^2}{\hat{a}_v^2}\right\}$$

$V_{50}$ = 50-year return wind speed

$\hat{a}_v$ = $4\sigma_v$ and $\sigma_v$ = rms moment corresponding to the wind speed $V$

$\sigma_v$ = $\sigma_{v_c} K^{3/2} \exp\left\{-\frac{1}{2}\left(\frac{1-K^{-1}}{B}\right)^2\right\}$

$D(a)$ = damage due to one cycle with a moment amplitude $a$.

The above relationship yields the damage $\bar{D}$ as a function of $\theta = V_c/V_{50}$ and the peak stress, $\hat{f}_{v_c}$, corresponding to the peak moment amplitude $\hat{a}_{v_c}$ developed at the critical speed. The value of $\hat{f}_{v_c}$ necessary to achieve a desired fatigue life $T$ can then be computed as a function of $\theta$ . Computations of the type outlined above were done for both concrete and steel and the resulting relationships between fatigue life and maximum stress at critical speed were fitted by empirical relationships which were;

(i) for the reinforcing steel

$$f_{max} = \frac{5}{3-2R}\left\{115 - 15\log_{10} f_o T + \left(360 - 100\log_{10} f_o T\right)\left|\frac{\theta-0.25}{0.75}\right|^{2.7}\right\}$$

$f_o$ = natural frequency (Hz);

$T$ = fatigue life (years)

$f_{max}$ = max tensile stress in MPa at the critical speed and with a peak factor ($\hat{a}/\sigma$) of 4

$R$ = $f_{max}/f_{min}$ (tension positive)

$\theta$ = $V_c/V_{50}$ (both evaluated at the same position)

(ii) for the concrete

$$\frac{f_{max}}{kf_c'} = 0.49 - 0.06\log_{10} f_o T + 0.31\left(\frac{\theta-0.25}{0.75}\right)^2$$

The allowable steel stress for $R$=-1 (complete reversal) is shown in Fig. 3 for $f_o$ = 0.3 at $T$ = 10, 20, 50, 100 and 200 years ie. $f_o T$ = 3, 6, 15, 30 and 60. The allowable concrete stress ratio ($f_{max}/k f_c'$ is shown in Fig. 4 for $f_o T$ = 1, 30 and 1000. In the case of values of $\theta > 1$ the steel stress is evaluated at $V = V_{50}$ and the allowable value is that shown for $\theta$ = 1. For concrete, the stress is calculated for $V = V_c$ and the allowable value at the relevant $\theta$ .

## 4. SENSITIVITY STUDY

While it is not immediately apparent from the results presented here it is the limiting steel stress which will normally be the controlling factor and the sensitivity study was limited to this stress. Fig. 5 shows the effect of bandwidth ($B$) variation and wind climate (exponent $\gamma$ ) while Fig. 6 shows the effect of changing from a uni-directional to a uniform omni-directional climate. In both figures the comparison is made for $f_o T$ = 15.

## References:

[1] Helgason, T. et al. "Fatigue strength of high yield reinforcing bars", Nat. Co-op Hwy. Res. Prog. Rep. No. 164, Trans. Res. Board, Washington, DC, 1976.

[2] Kakuta, Okamura & Kohno "New concepts for concrete fatigue design procedures in Japan", Proc. Int. Ass. Br. & Struct. Eng'g Colloquium, Lausanne, 1982, IABSE Rep. Vol. 37, pp. 51-58.

[3] Daly, A. "The response of chimneys to wind-induced loads and the evaluation of the resulting fatigue damage", Ph.D. Thesis, U.W.O., London, Canada, 1986.

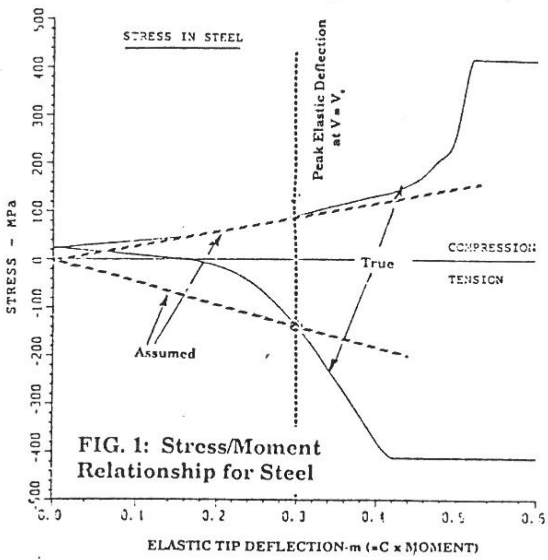


FIG. 1: Stress/Moment Relationship for Steel

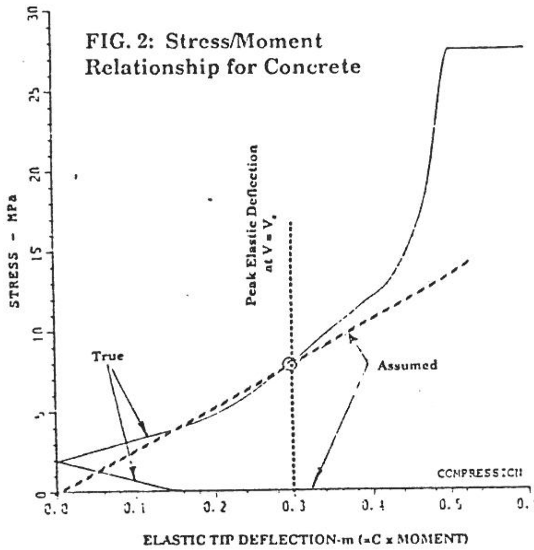


FIG. 2: Stress/Moment Relationship for Concrete

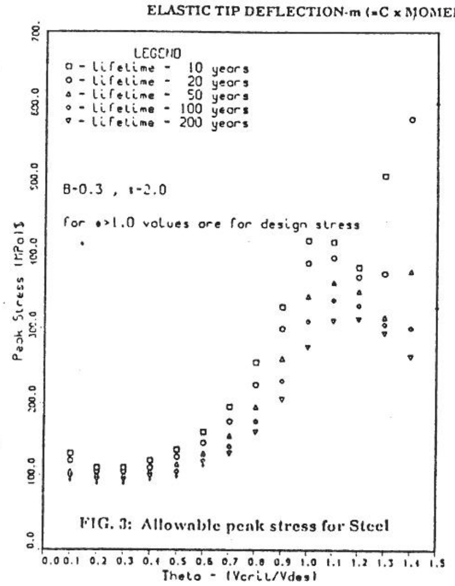


FIG. 3: Allowable peak stress for Steel

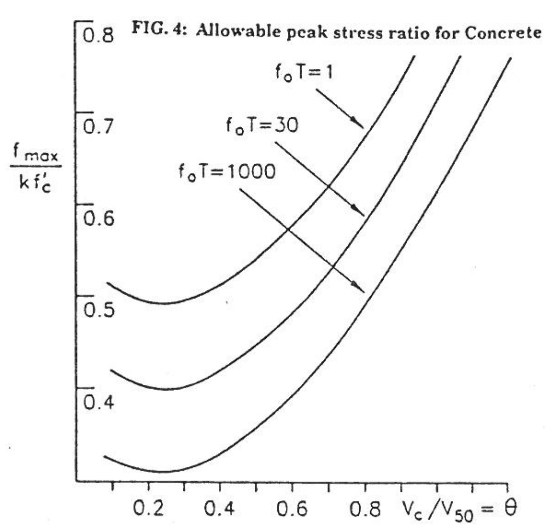


FIG. 4: Allowable peak stress ratio for Concrete

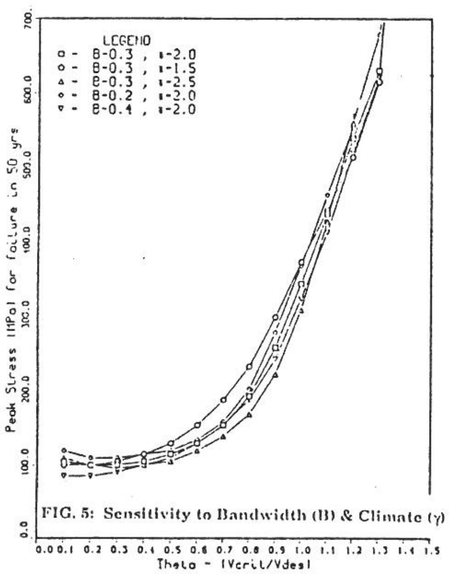


FIG. 5: Sensitivity to Bandwidth (B) & Climate (γ)

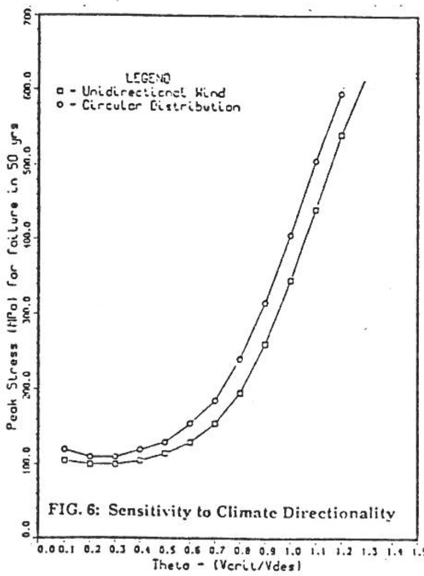


FIG. 6: Sensitivity to Climate Directionality