Kwok

ON THE MAXIMUM OVER-PRESSURE RESULTING FROM SUDDEN WINDOW OPENING

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SUMMARY

Previous studies have examined the maximum over-pressure that can result from sudden window opening that might occur as a result of a breakage during a windstorm. A practical limitation of these investigations [Ref.2] has been that the peak internal pressure developed during such an event has to be computed numerically because of the non-linear differential equation governing the internal pressure fluctuations.

The following paper derives deterministic formulae for the peak internal over-pressure, instantaneous peak flowrates and total first cycle inflow during such a sudden window opening. The formulae only require as input the internal volume, size of vent opening and the initial pressure difference across the opening.

INTRODUCTION

The equation of motion governing unsteady flow through the opening of an enclosed (i.e. internal) volume has been previously documented [Refs.1,2,3] and can be written as:

$$\frac{d^2p_i}{dt^2} + \frac{1}{2}.\phi.\frac{dp_i}{dt}\left|\frac{dp_i}{dt}\right| + w_o^2.p_i = w_o^2.p_{ext}$$

$$Q = \beta.\frac{dp_i}{dt}$$

In the above, p_i = internal pressure, p_{ext} = external pressure and Q = flow through the opening. The constants in the above equations are:

$$\phi = \frac{V_o}{C_d^2 k p_a L_e A_o}$$

$$w_o^2 = \frac{k p_a A_o}{\rho L_e V_o}$$

$$\beta = \frac{V_o}{k p_a}$$

where ... V_o = internal volume, A_o = area of opening, C_d = opening discharge coefficient, k = specific heat capacity ratio, L_e = air slug effective length, ρ = air density and p_a = atmospheric pressure. For most applications we can assume typical values for some of the above constants as ρ = 1.2 kg/m^3, p_a = 101.3 kPa, k = 1.4 (adiabatic) and for typical opening geometries, C_d = 0.6 and $L_e = 0.9\ A_o^{0.5}$.

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Undamped Solution

With no damping the solution to the above differential equation governing the fluctuations of the internal pressure yields the well known Helmholtz Resonator circular frequency, w_o. If we assume the previous "typical" constant values, the undamped natural frequency is given approximately by:

$$f_o = 57.7 \sqrt{\frac{A_o^{0.5}}{V_o}}$$

For a room with a volume of $V_o = 200\ m^3$ and for a reasonably large opening of $A_o = 2.0\ m^2$, the internal pressure fluctuations would have a period of about 0.21 seconds.

Damped Solution:

We now seek a solution to the original differential equation with the non-linear damping term included. We firstly re-write the equation with $p = p_{int} - p_{ext}$, and hence seek to solve:

$$\frac{d^2p}{dt^2} + \frac{1}{2}.\phi.\frac{dp}{dt}\left|\frac{dp}{dt}\right| + w_o^2.p = 0$$

By making the following substitutions:

$$\frac{dp}{dt} = x$$
$$f = x^2$$

the original non-linear differential equation can be separated into two first order, linear differential equations, namely:

$$\frac{df}{dp} + \phi.f = -2w_o^2p$$

$$\frac{df}{dp} - \phi.f = -2w_o^2p$$

which apply when $x > 0$ and $x < 0$ respectively. The solutions to these two "branches" of the original differential equation are:

$$f = \frac{2w_o^2}{\phi^2}(1 - \phi p) + A_1.e^{-\phi p}$$

$$f = \frac{2w_o^2}{\phi^2}(1 + \phi p) + A_2.e^{\phi p}$$

The motion described by these two equations is oscillatory, damped, with alternative solutions given as the gradient of p, i.e. the flow into and out of the internal volume, changes from positive to negative. The constants, A_1 and A_2, in the above equations depend upon the initial conditions. Suppose we have:

At time $t < 0$:			At time $t = 0$:		
	p_{ext}	$= 0$		p_{ext}	$= 0$
	p_{int}	$= -P_o$		p_{int}	= can fluctuate
	A_0	$= 0$		A_0	$= A_o$
(i.e. vent opening is closed)			(i.e. sudden opening)		

During the first internal pressure cycle, the flow will be INTO the volume, and the internal pressure will reach some maximum value, equal to $+P_1$, the "over-pressure". Substitution of the initial values into the positive branch solution of the above equations yields the following relationship between the initial pressure difference set up across the opening and the first maximum peak sustained by the internal volume:

$$\frac{1+\phi P_o}{e^{\phi P_o}} = \frac{1-\phi P_1}{e^{-\phi P_1}}$$

The above can be solved iteratively. However, an approximate solution which is accurate for all values of ϕP_o to within +/−2% is:

$$\frac{P_1}{P_o} = \lambda_1 = \frac{1-e^{-\phi P_o}}{\phi P_o}$$

The equations can be further analysed to yield the following additional useful formulae:

Maximum First Cycle Instantaneous Flowrate:

During the first internal pressure cycle, when air moves INTO the internal volume, there is a point at which the flow is at a maximum. This maximum flowrate, Q_{max}, is:

$$Q_{max} = C_d A_o \sqrt{\frac{2P_o}{\rho}} \cdot \sqrt{\lambda_2}$$

$$where\ \lambda_2 = (1 - \frac{\ln[1+\phi P_o]}{\phi P_o})$$

Total First Cycle Inflow:

During this same first fluctuation cycle, the total amount of air, W_{total}, that enters the internal volume is given by:

$$W_{total} = (1 + \lambda_1)\ \beta\ P_o$$

SIMPLIFIED FORMULAE:

To make the computations somewhat more amenable, the above equations were simplified by assuming the previous typical values for all the equation constants leaving only three remaining variables:

Input Variables: $\mathbf{V_o, A_o, P_o}$

Working Parameters:

$$\alpha = \frac{P_o V_o}{46{,}000 A_o^{1.5}}$$

$$\lambda_1 = \frac{1 - e^{-\alpha}}{\alpha} \ , \ \lambda_2 = 1 - \frac{\ln(1+\alpha)}{\alpha}$$

$$Q_o = 0.77\, A_o \sqrt{P_o} \ , \ W_o = \frac{P_o V_o}{142{,}000}$$

Computed Parameters:

$$P_1 = \lambda_1 P_o \quad \textit{"maximum pressure"}$$

$$Q_{max} = \sqrt{\lambda_2}\, Q_o \quad \textit{"maximum flowrate"}$$

$$W_{total} = (1 + \lambda_1)\, W_o \quad \textit{"total inflow"}$$

Example Computation:

Assume as input data $V_o = 200\ m^3$, $A_o = 2.0\ m^2$, and let $P_0 = 1000$ Pa:

Hence: $\alpha = 1.537$ $\lambda_1 = 0.51$ $\lambda_2 = 0.39$
$Q_o = 48.70$ $W_o = 1.41$

And ... $P_1 = 510.7$ Pa $Q_{max} = 30.6\ m^3/s$ $W_{total} = 2.13\ m^3$

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