GLASS STRENGTH UNDER WIND LOADING

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SUMMARY A new formulation to approximate the non-linear relationship between load and stress, has been used to derive the probability distribution of breakage pressure for a glass panel from the probability distribution of crack depth. Using the new formulation, the authors have found that the glass strength charts published in ASTM1300 may not be conservative.

1 INTRODUCTION

Since the strength of the glass varies with load duration, and fluctuating pressures which load the window panes are continually being applied during a wind storm, the cumulative effect of the wind pressure fluctuations over the entire duration of the wind storm needs to be computed. Research is currently being carried out Monash University in order to establish an appropriate method for determining the equivalent design wind pressure for window glass design. The results reported in this paper are part of this current research work.

2 THE STRENGTH OF GLASS

The inherent weakness of glass was first explained by Griffith (1). He postulated that the presence of minute surface cracks caused stress concentrations and he devised a theory capable of predicting the strength of glass in relation to the size of the cracks. The existence of surface flaws also explained the variability which is observed in the strength of glass. The magnitude of the applied stress that will cause failure depends on the size of the flaw, and this is a matter of probability. It has been postulated that the variation of strength with load duration (called "static fatigue") is due to the presence of water vapour, which enters the tiny surface cracks and weakens the glass by chemical attack at the crack tip. (2)

Theoretical and empirical relationships to model Static Fatigue have been developed by various researchers. Brown (3) gave an expression, for integration of stress-time effects for any specific flaw which leads to failure, which for constant humidity and temperature can be simplified to:

$$C = \int_0^{t_f} [\sigma(t)]^n dt = constant \tag{1}$$

3 STRESSES DEVELOPED IN GLASS PANELS SUBJECTED TO UNIFORM PRESSURE

Glass behaves in an elastic manner and obeys Hooke's law, right up to the instant of fracture. However, simple bending theory is not valid when the deflection of the panel is more than half the panel thickness. Since panels of glass are usually very thin, relative to their area, the deflections at breakage loads are usually much greater than the glass thickness and consequently, due to the establishment of in-plane (membrane) stresses the relationship between applied load and stress (and deflections) becomes non-linear.

A number of computer programs have been written since the early 1970's. The results from these computer programs have shown that the relative distribution of the total tensile stress on the surface (which is the sum of the bending and membrane stresses) varies as the applied load is increased. At low loads the membrane stresses are negligible and simple bending theory may be used to give the same results. At high loads the membrane stresses become significant and the maximum stresses lie at some point on the diagonal, moving closer to the corner of the panel as the load increases.

A series of curves (ESDU 71013) to enable determination of stresses and deflections of rectangular plates under uniformly distributed normal pressure was published in 1971 by the Engineering Sciences Data Unit, Royal Aeronautical Society London. These curves, which are based on the non-linear theory, agree closely with experimental results and the theoretical results of other researchers.

A comparison of the stresses obtained using ESDU 71013 with the stresses measured by researchers at the Ontario Research Foundation (ORF), on a panel of 6 mm toughened glass of size 1525 mm x 2440 mm, was made by Calderone (4). This comparison (see Figure 1) showed that there was excellent agreement between the stresses measured along the diagonal and the ESDU 71013 diagonal stresses. There was also close agreement between the centre stresses, although agreement was better at the lower loads.

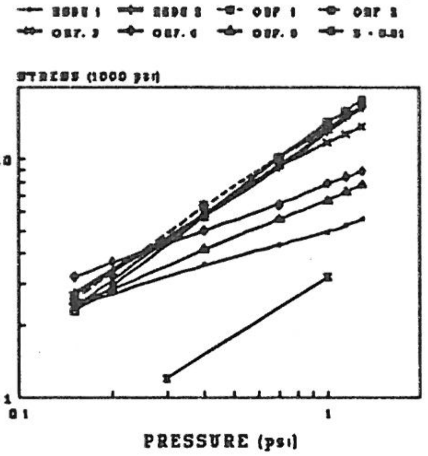

Figure 1. O.R.F. Stress Measurements Compared to ESDU Stress Versus Load (or uniform pressure).

In this figure, ESDU 1 and 2 are the stresses at the centre and at the diagonal, calculated using the ESDU charts and ORF 1, 2, 3, 4 and 5 are from strain gauge measurements at positions on the diagonal, 54, 216, 431 and 700 mm from the corners and at the centre of the panel respectively. It can also be seen from Figure 1 that the variation of stress with applied load, is different at different points on the glass panel and that the relationship between load and stress can be approximated by straight lines on this figure, which is drawn using double logarithmic scales.

Consequently, Calderone (15) assumed that the variation of stress versus load can be approximated by a relationship of the form:

$$\sigma = a\,P^{S} \qquad (2)$$

Thus, if the variation of pressure with time P(t) is known, then the variation of stress with time, (t), can be determined and the cumulative damage criterion of equation (1) can be written, for a particular panel, as:

$$\int_0^{t_f} [P(t)]^{S.n}\, dt = constant \qquad (3)$$

It should be noted that the constants, a and S in equation (2), vary with location on the panel. It has been postulated that for any particular panel there will be only one value of S that is critical and if this is valid, then it will be possible to more readily obtain equivalent pressures, from wind tunnel measurements, for glass design.

The possibility of obtaining an effective value of S was then examined by Calderone (4), using the results of breakage tests conducted by the Ontario Research Foundation. These tests were on samples of 6 mm Float glass of size 1525 mm x 2440 mm and various loading rates were used to achieve fracture of a large number of panels. The breakage pressures and time taken to fracture for each test were reported by Johar (16) and Calderone calculated the mean breakage pressure and the mean time to breakage for each loading rate used. A line of "best fit" was then determined using geometric regression analysis and the results are shown in Figure 2.

The slope of this line was found to be -12.96 and this indicates that the cumulative damage criterion for this size and thickness of glass panel is given by:

$$\int_0^{t_f} [P(t)]^{12.96} dt = constant \qquad (4)$$

That is, the product $S.n$ in equation (3) equals 12.96. Consequently, taking n=16, the value of S for this panel is 12.96/16 = 0.81. This slope was drawn on Figure 2 and was found to be within the slopes of the measured stresses which occurred over the surface of the panel. It therefore appeared to Calderone (4) that it may be possible to determine an effective value of S for a particular panel, enabling the cumulative damage criterion to be approximated by equation (3).

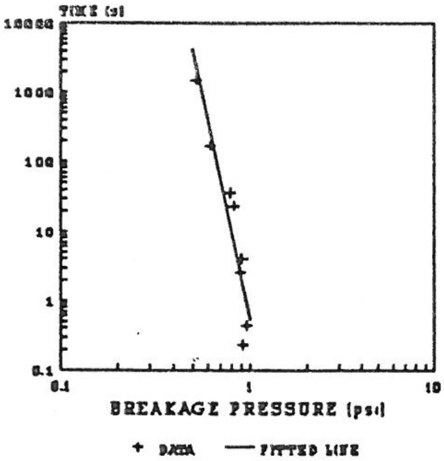

Figure 2 Time to Breakage versus Breakage Pressure (from ORF tests).

4 GLASS STRENGTH DATA

For a linearly increasing load, it can be shown, from equations (1) and (2), that the equivalent constant stress and the equivalent constant pressure are given by the following expressions:

$$\sigma_e = \sigma_f \left(\frac{t_f}{t_e (Sn+1)} \right)^{\frac{1}{n}} \qquad (5)$$

$$P_e = P_f \left(\frac{t_f}{t_e (Sn+1)} \right)^{\frac{1}{Sn}} \qquad (6)$$

The results reported by Johar (5) were converted (equivalent to 60 second load duration) using equations (5) and (6). The values of the constants, S and a used, being estimated from the strain gauge measurements, of the maximum principal tensile stress nearest to each fracture origin. This converted stress was then used to calculate the crack depth at each fracture origin. The distributions of crack depth, maximum principal tensile stress, and pressure at breakage are given in figures (3), (4) and (5).

Wadsworth (6) states that a lognormal distribution has been used, to model stress failure mechanisms, when a crack in the structure has reached a given size and the growth of the crack, at any instant is a random proportion of its size at that time. Therefore, a lognormal distribution was determined, for the above results and this is also shown in figures (3), (4) and (5).

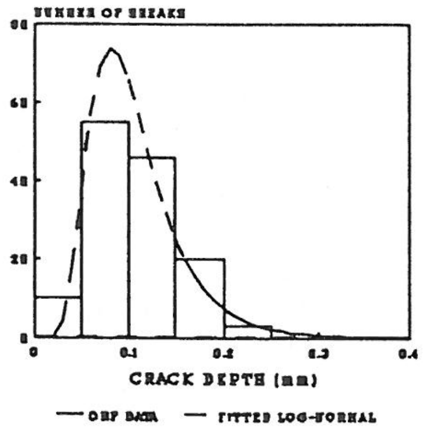

Figure 3 Distribution of crack depth determined from ORF data.

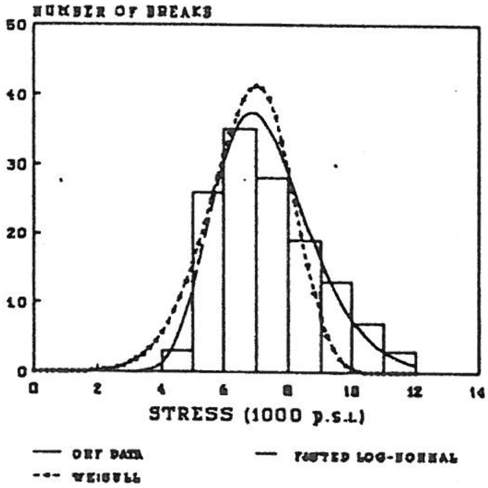

Figure 4 Distribution of stress at breakage from ORF data.

From figures (3), (4) and (5), it appears that a lognormal distribution represents the data well (and better than a Weibull see figure (4)). Consequently, if an effective value of the constants S and a is valid for the panel as a whole, then these constants can be found as follows.

The density function for the lognormal distributions of breakage pressure and stress are given by:

$$f(FS) = \frac{\exp\left(-\frac{1}{2}\left(\frac{\ln(FS)-\mu_{FS}}{\sigma_{FS}}\right)^2\right)}{\sqrt{(2\pi)}\,\sigma_{FS} FS} \qquad (7)$$

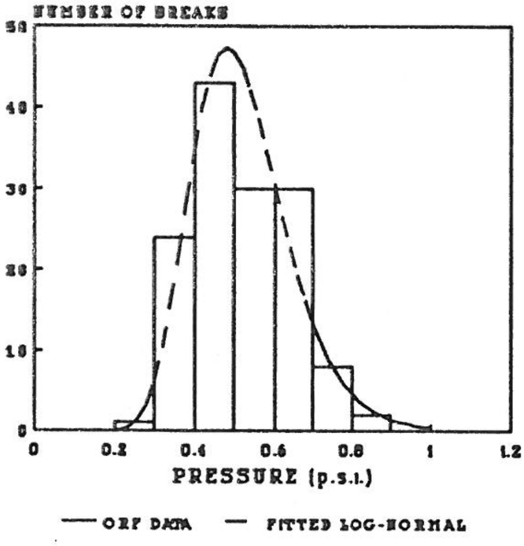

Figure 5 Distribution of breakage pressures from ORF data.

$$f(P)=\frac{\exp(-\frac{1}{2}(\frac{\ln(P)-\mu_P}{\sigma_P})^2)}{\sqrt{(2\pi)}\sigma_P P} \qquad (8)$$

Now, substituting equation (2), it can be shown that:

$$\sigma_P = \frac{\sigma_{FS}}{S} \qquad (9)$$

$$\mu_P = \frac{(\mu_{FS}-\ln(a))}{S} \qquad (10)$$

Hence, from the lognormal distributions shown in figures (4) and (5) and equations (9) & (10), it was found that S=0.89, which is reasonably close to the value of 0.81 found previously, by plotting the breakage rates (see figure (2)). The difference in the results is probably due to the inaccuracies of the estimates made of the values of S and a at each fracture location.

In a similar manner, the results obtained by Beason, which were for glass removed from a 20 year old building, were converted to the equivalent for 60 second load duration using equations (5) and (6). In this case, since strain gauge measurements were not made in these tests, the constants, S and a used, were estimated from the ESDU charts. The distribution of breakage pressures for these results is shown in figure (6). It was found that the distribution of breakage pressure for the inside surface was different to that of the outside surface. However, Beason used the combined data for his analysis of glass strength.

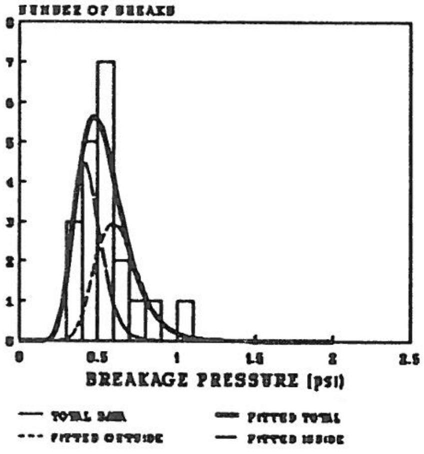

Figure 6 Distribution of breakage pressure from Beason's tests.

Finally, the surface strength parameters (mean and standard deviations of the lognormal distributions) found from Beason's results, were used with the glass geometry parameters (S and a) for the glass size used in the ORF results, to find the distribution of breakage pressure, for old glass of the size used on the ORF tests. (See figure (7).)

5 DISCUSSION

From figures (4) and (5), and equations (7) to (10), it can be seen that it is possible to determine the distribution of breakage pressure for a glass panel, from the distribution of breakage stress (or the crack depth), provided the glass geometry parameters (S and a) can be determined for the panel as a whole. Using estimated parameters for the panels of glass tested by Beason, the distribution of breakage stress was found for the 20 year old glass tested and using this distribution with the parameters for the sizes tested by ORF gave a probability of breakage of 2% at the allowable design wind pressure given by ASTM1300 for this size of glass. However, ASTM1300 states that the probability of breakage is 8 in 1000. Consequently, the probability of breakage of the ASTM1300 charts may be higher than it states.

The results presented in this paper are preliminary only. Therefore, it is necessary to carry out more detailed analyses using more accurately determined parameters (S and a) and further testing on older glass should be carried out to determine the appropriate distribution of crack depth for design of window glass panels.

6 CONCLUSION

It has been found that, a lognormal probability distribution, gives a much closer representation of the glass breakage data than a Weibull distribution. Also, by using a new formulation to approximate the non-linear relationship between load and stress, it is possible to derive the probability distribution of breakage pressure for a glass panel from the probability distribution of crack depth. Using this method allows convenient analysis of the probability of breakage of glass panels, provided suitable glass geometry parameters can be determined.

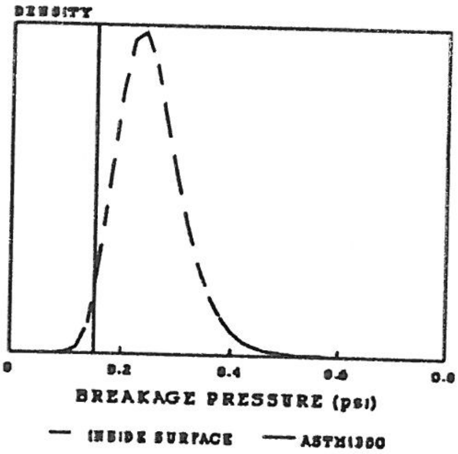

Figure 7 Distribution of breakage pressure for old glass in ORF test size panels.

7 REFERENCES

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