WAS THE HARRIS AND DEAVES VELOCITY PROFILE A STEP IN THE WRONG DIRECTION?

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INTRODUCTION

Successful and accurate predictions strong winds and the damage that they cause have great practical importance. This paper is about findind a velocity profile for strong winds generated by upper level pressure gradients.

The Harris and Deaves (HD) mean velocity profile [1] for strong winds is:

$$V = \frac{u_*}{\kappa}\left[\ln\frac{z}{z_0} + 5.75\left(\frac{z}{h}\right) - 1.88\left(\frac{z}{h}\right)^2 - 1.33\left(\frac{z}{h}\right)^3 + .25\left(\frac{z}{h}\right)^4\right] \qquad (1)$$

V is the mean velocity at height z. u_* is the friction velocity. $\kappa = 0.41$ is Von Karman's constant. z_0 is the ground roughness length. This formula has been incorporated in the Australian and New Zealand wind loading codes.

No published description of the derivation of the HD velocity profile is known. The derivation below was given to me by Professor Harris.

DERIVATION

From similarity theory we have:

$$\kappa V / u_* = \ln z / z_0 \qquad \text{for } z \text{ small} \qquad (2)$$

$$h = u_* / \beta f \qquad (3)$$

$$\kappa G / u_* = \ln(u_* / f z_0) - A = \ln(h / z_0) + \ln\beta - A \qquad (4)$$

A and β are constants to be found from experimental data. As z approaches h, we expect:

$$\kappa(V - G)/u_* = F(z/h) \text{ with boundary conditions } V = G \text{ and } dV/dz = 0 \text{ at } z = h \qquad (5)$$

From the low-level form of $\kappa V/u_*$ given in (2) and the expression for $\kappa G/u_*$ given in (4), the only possible profile law for V throughout the boundary layer which will satisfy (5) is given by the form:

$$\kappa V / u_* = \ln z / z_0 + \phi(z/h) \qquad (6)$$

where $\phi(0) = 0$ in order to satisfy (1) for z small (7)

From (4), (5) and (6):

$$\kappa(V - G)/u_* = F(z/h) = \ln(z/h) + \phi(z/h) - \ln\beta + A \qquad (8)$$

We now introduce the new variable $\eta = 1 - (z / h)$ so that in this new variable the top boundary condition becomes $V = G$ and $dV/d\eta = 0$ at $\eta = 0$ and we have:

$$\kappa(V - G)/u_* = \ln(\eta) + \Phi(\eta) - \ln\beta + A \qquad (9)$$

Measurements suggest a parabolic velocity defect law for about the upper 70% of the boundary layer. As η increases from zero, the series for $\ln(1-\eta)$ converges only slowly, so that we can only obtain a parabolic velocity defect law for values of η up to about 0.7 if the function $\Phi(\eta)$ is such that the terms in its power expansion cancel those in the expansion of $\ln(1-\eta)$, except the term in η^2. We arbitrarily chose to truncate the expansion at the term in η^4, so that the departures from a parabolic velocity law are $O(\eta^5)$ which guarantees about 1% error up to amount $\eta = 0.7$ for the parabolic velocity defect law (clearly higher order terms could be included). Thus HD obtained:

$$\Phi(\eta) \approx -A + \ln\beta + \eta + \alpha\eta^2 + \eta^3/3 + \eta^4/4 \qquad (10)$$

where $\alpha = A - \ln\beta - 1 - 1/3 - 1/4$ in order to satisfy the boundary condition (7) which becomes $\Phi = 0$ for $\eta = 1$ in terms of the new variable η.

Using this expression for $\Phi(\eta)$ and restoring the original variable z/h leads to:

$$\frac{\kappa V}{u_*} = \ln\left(\frac{z}{z_0}\right) + \left(\frac{1}{6} + 2\ln\beta - 2A\right)\frac{z}{h} - \left(A - \ln\beta + \frac{11}{12}\right)\frac{z^2}{h^2} - \frac{4}{3}\frac{z^3}{h^3} + \frac{z^4}{4h^4} \qquad (11)$$

Experimentally one finds $A = -1$, $\beta = 6$, thus:

$$\frac{\kappa V}{u_*} = \ln\left(\frac{z}{z_0}\right) + 5.75\left(\frac{z}{h}\right) - 1.88\left(\frac{z}{h}\right)^2 - 1.33\left(\frac{z}{h}\right)^3 + 0.25\left(\frac{z}{h}\right)^4 \qquad (12)$$

PROBLEMS STEMMING FROM THE DERIVATION

Eq. 12 is not the only profile that satisfies the boundary conditions in equations 2, 4 and 5 as well as the parabolic velocity defect law in the upper 70% of the boundary layer when $A = -1$ and $\beta = 6$. Two alternative formulae that satisfy all the conditions are plotted along with the HD formula in Figure 1. These alternatives were created using a blending technique that uses the tanh function [2].

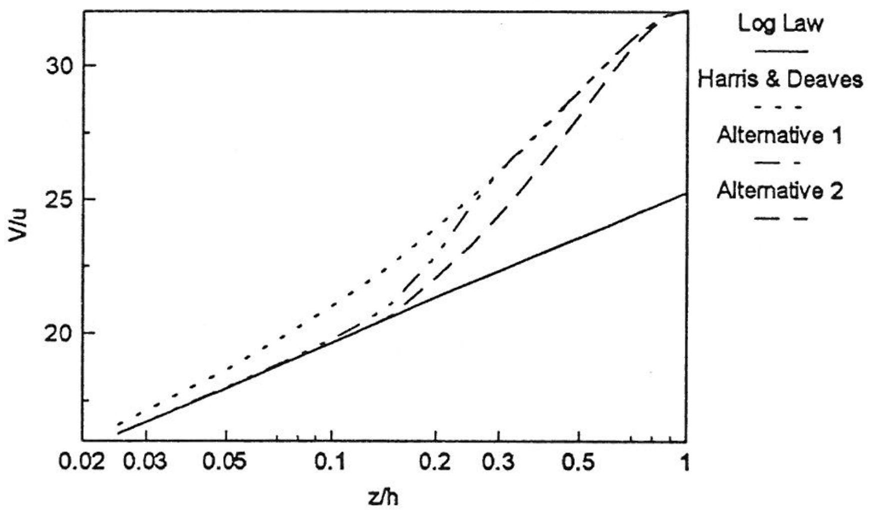

Figure 1. The non-uniqueness of formulae that satisfy the HD boundary conditions.

The one thing that sets the HD formula apart from all others is the use of the power series expansion for $\ln(1-\eta)$ about $\eta = 0$. This ensures good agreement with the parabolic velocity defect law near the top of the atmospheric boundary layer but gives a very poor formula in the bottom 20% of the boundary layer where accuracy is most needed.

The near ground part of the power series is very inaccurate and this is used in conjunction with the ground boundary condition (7) in the evaluation of α. This means that the value of α in the above derivation is inaccurate. The curvature of the parabolic velocity defect law at the top of the boundary layer is directly related to α. Because this curvature is wrong the upper part of the velocity profile is inaccurate. The net result is that the whole of the HD velocity profile is inaccurate.

The values of A and β used by HD are inaccurate. A review by Zilitinkevich [3] suggests $1/\beta = 0.3$. The value of A, calculated from A_0 and B_0 in this review by:

$$(11.56 - B_0)^2 + A_0^2 = (11.56 + A)^2 \qquad A_0 = 4.5 \qquad B_0 = 1.7 \qquad (13)$$

is $A = -0.72$. A typical value for $\ln(h/z_0)$ of 10.36 is used in this derivation.

COMPARISON WITH OTHER BOUNDARY LAYER FORMULAE

One of the earliest and best known formulae is the Power law formula:

$$V / u_* = C(z / z_0)^{1/n} \tag{14}$$

This formula does not satisfy Eq. (2) but has been found to agree fairly well with experimental data. Another early formula is the Log law profile:

$$V = \frac{u_*}{\kappa} \ln \frac{z}{z_0} \tag{15}$$

Coles' [4] velocity profile was developed in 1956 for laboratory boundary layers and is still widely used. Coles' mean velocity profile is based on Eq. (6) and can be written:

$$V = (u_* / \kappa)(\ln z / z_0 + \Pi w(z / h)) \tag{16}$$

Π is a profile parameter and w is the wake function. No attempt is made to force this equation to match a parabolic velocity defect law. The wake function w is found from a large amount of experimental data and is published by Coles in tabular form.

The wake function can be approximated by:

$$w(\zeta) = 1 - \cos(\pi\zeta) \tag{17}$$

The value for profile parameter Π to match A and β derived from the review by Zilitinkevich is $\Pi = 0.96$. The Coles' mean velocity profile need not satisfy Eq. (5) because the experimental data from which the profile has been derived supports the contention that dV/dz is discontinuous at $z = h$.

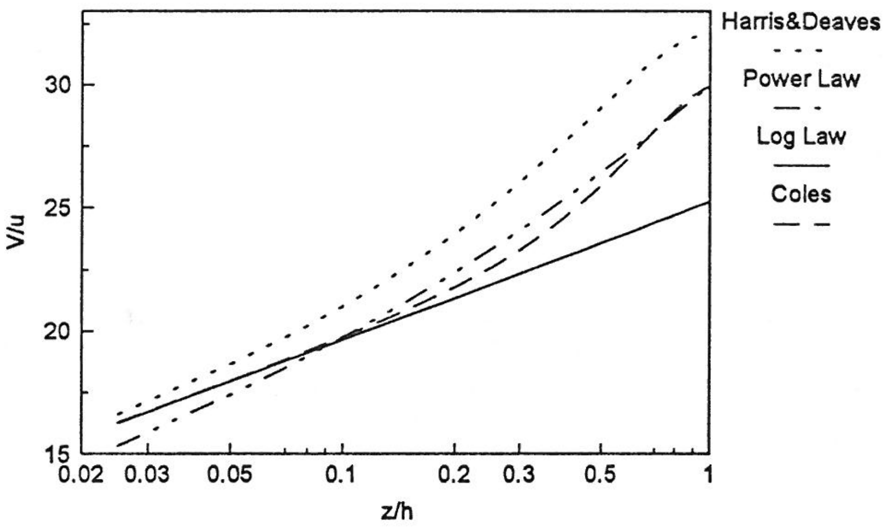

Figure 2. A comparison of velocity profiles.

Eqs (1), (14), (15) and (16) are compared in Fig. 2. The value $\ln(h/z_0)$ is set to a typical value of 10.36 to allow direct comparison.

The HD profile does not satisfy Eq. 2; it does not behave like the logarithmic law of the wall at small values of z. This is because the HD profile has a linear deviation from the Log law near the ground. This linear deviation near the ground is a feature of non-neutral boundary layer velocity profiles.

Practical applications of mean wind flow profiles usually involve the lower 20% of this profile. Profiles are matched to the value and slope of the Log law profile at 5% of the boundary layer height and deviations at 10% and 20% are noted. A typical value for $\ln(h/z_0)$ of 10.36 is used.

Table 1. Deviation from the Log law in the lower 20% of the boundary layer.

	z/h = 0.05	z/h = 0.1	z/h = 0.2
Log Law Profile	0	0	0
Power Law Profile	0	0.08	0.34
Coles Profile	0	0.08	0.41
Harris & Deaves Profile	0	0.15	1.76

The HD profile differs much more from the Log law profile than the Power law profile does in the lower region of the boundary layer. To make the HD profile match the Log law profile at z/h = 0.05 requires a massive change of 28% in u_* and 484% in z_0.

COMPARISON OF PROFILES WITH DATA FROM HARRIS & DEAVES [1]

HD report the agreement between their profile and data from Nantes, Rugby, Cranfield, Leipzig and Farnborough. The Nantes and Cranfield results only extend up to z/h=0.04, at which point any deviation from the Log law is negligible. The Rugby, Leipzig and Farnborough results are plotted in Figure 3.

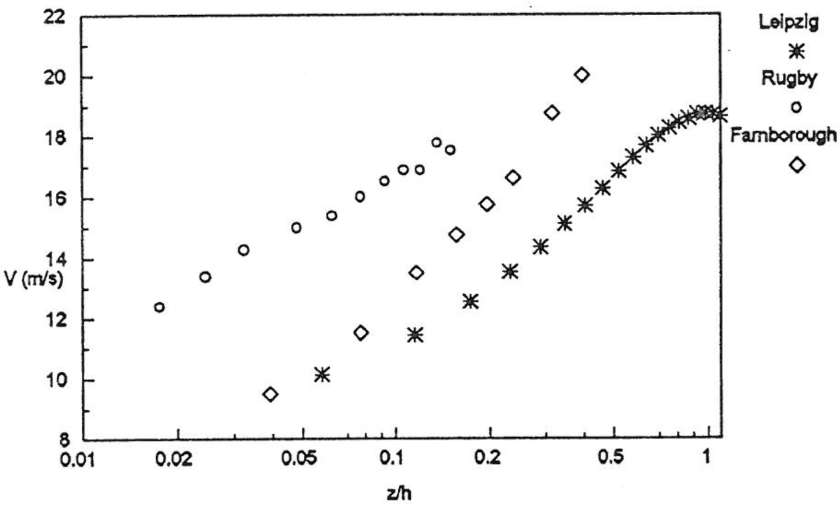

Figure 3. Data from Harris and Deaves.

The reported G value [5] is used for the Leipzig profile and an arbitrary value of G = 25 m/s is used for scaling the other data. The Rugby data shows no deviation from the Log law. The Farnborough data shows a deviation from the Log law that is of the same order of magnitude as Coles profile or the Power law profile. This deviation is smaller than could be explained by the HD profile (see Table 1).

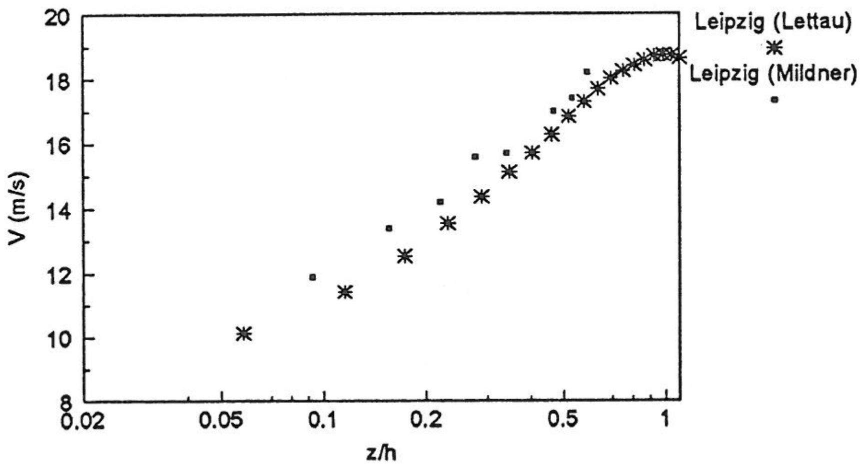

Figure 4. Two interpretations of the velocity data from Leipzig.

The Leipzig data agrees very well with the HD profile and not at all well with the other profiles. It does not match the Log law at all, even at small z/h. The Leipzig profile was derived by Lettau [5]. It is a semi-theoretical profile based on original data reported in 1932 by Mildner [6]. This data is plotted in Figure 4.

The peak velocity in Mildner's data is 20.9 m/s which occurs at an unstated height. Mildner's data shows a much smaller deviation from the Log law than Letau's profile. This deviation is of the same order of magnitude as in the Coles profile or the Power law profile. It is smaller than could be explained by the HD profile.

COMPARISON OF PROFILES WITH DATA FROM OTHER SOURCES.

Fig. 5 shows data collected at Rugby (obtained privately). The winds are fairly strong. The profiles were measured by anemometers mounted on a mast surrounded by flat terrain. The highest anemometer was 182m above ground level. This has a z/h value of about 0.153 if the HD estimate of β = 6 is used. My detailed statistical analysis shows that there is a slight deviation from the Log law. The Power law fits the data better than the Log law for all but two of the 11 profiles (Runs 70 and 87). The HD profile does not fit the data best for any of the 11 runs.

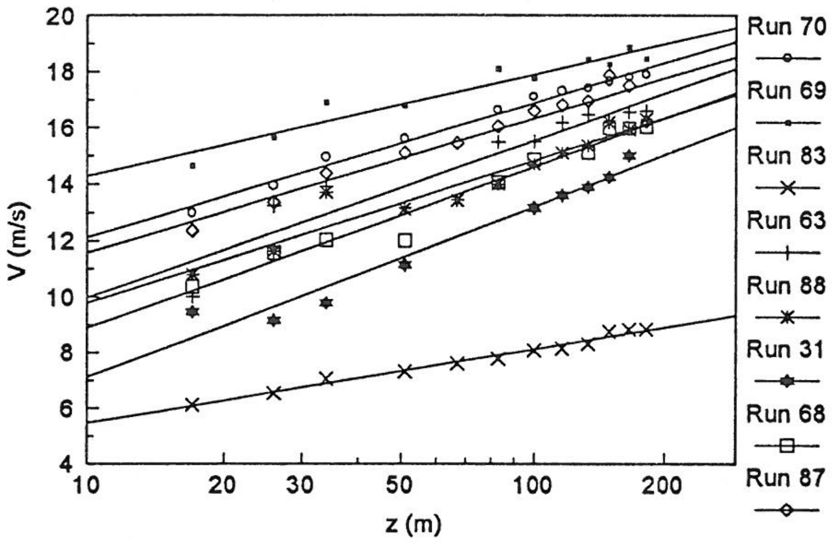

Figure 5. Strong wind data from Rugby.

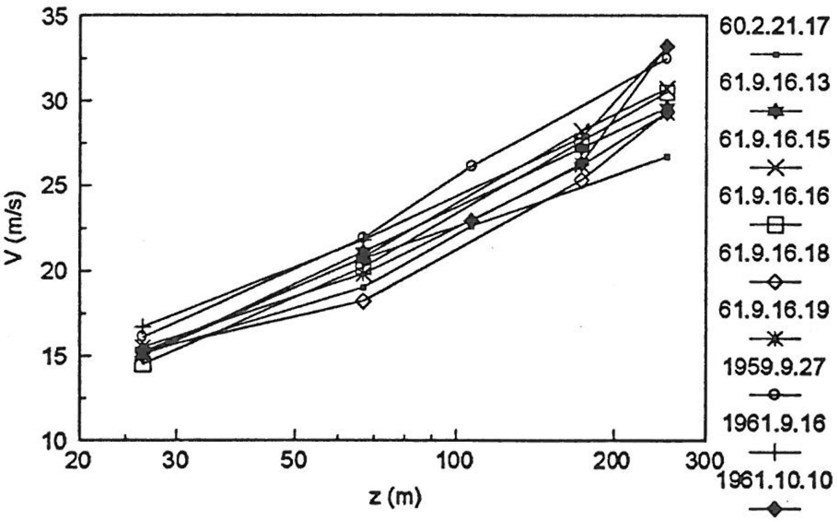

Figure 6. Strong wind data from Tokyo Tower

Soma [7] reported wind profiles to heights of 253 m and velocities to 33.2 m/s measured at Tokyo Tower. This data contains some cyclone winds and some ordinary winds. Nine of these profiles, selected for high wind speeds at the highest and lowest anemometer, are shown in Figure 6. This figure does not include data from the anemometer at 107 m in the six profiles measured on the same day (61.9.16) because it consistently reads low on that day.

These nine profiles show some deviation from the Log law profile. Four profiles most closely match the Log law profile, four most closely match the Power law profile and only one closely matches the HD profile. This is the profile 61.9.16.16.

CONCLUSIONS

The HD profile is not the only profile that satisfies the boundary conditions specified in its derivation. There are an infinite number of alternative formulae that satisfy these conditions. The one thing that sets the HD formula apart is the use of the power series expansion for $\ln(1-\eta)$ about $\eta = 0$. This gives a very poor formula in the bottom 20% of the boundary layer where accuracy is most needed. Because of the way that the lower and upper parts of the profile are linked through α, the curvature of the parabolic velocity defect law at the top of the boundary layer is inaccurate. The net result is that the whole of the HD velocity profile is inaccurate.

The HD profile does not behave like the logarithmic law of the wall near the ground because it has a linear deviation from the Log law at small values of z. The difference between the Power law and the Log law near the ground is much less than the difference between the HD formula and the Log law, even allowing for very large changes in u_* and z_0.

The HD profile agrees very poorly with measured data. It has worse agreement with data from Rugby, Tokyo Tower and data in HD own report than both Log law and Power law profiles.

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