A PROBABILISTIC MODEL OF SEVERE THUNDERSTORMS FOR TRANSMISSION LINE DESIGN

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Introduction

The design of transmission line structures is invariably governed by wind loading. The wind loads used in most design codes and standards for structures in Australia have been based almost exclusively on large scale wind storms (AS 1170.2–1989), including severe storms such as tropical cyclones. Very limited guidance is given on severe small scale (local) wind storms, such as severe thunderstorms. A recent review on failures of transmission line structures revealed that a large proportion of failures (more than 90%) are due to severe thunderstorms, such as tornadoes and downbursts. A brief overview of transmission line failures in many parts of the world shares the same conclusion.

As the number of transmission line failures increases, while more transmission lines are being built throughout the world, the costs associated with transmission line failures are being realised. It is therefore obvious that the necessity of design for severe thunderstorm events needs to be re-evaluated. The intention of this report is to propose a probabilistic model that could realistically and accurately estimate the design wind speed for transmission lines due to severe thunderstorms. Of particular interest are downbursts (or gust fronts) originating from thunderstorms since they are the main cause of transmission line failures in Australia. Based on the proposed model, a preliminary parametric study will be carried out with respect to different random variables involved.

This study is part of a larger study of thunderstorm winds and transmission line design, supported by ESAA (Electrical Supply Association of Australia).

Severe Thunderstorms

Thunderstorms have their genesis in the initial uplift of warm, moisture-laden air (see e.g. Li and Holmes, 1994). There are several different types of thunderstorms, depending on the origin and the associated meteorological activities. All types of thunderstorms can occasionally become severe. According to Australian climatology, a thunderstorm is considered severe if it produces winds in excess of 28.3 m/s (55 knots) (In the US, it is set at 25.9 m/s). The most severe thunderstorm is a tornado, which has been accorded sufficient attention in literature and therefore will not be discussed in the report. Another type of severe thunderstorm is the so-called downburst. A downburst is an intensive downdraft and gust front system. Downbursts can induce an outburst of damaging winds near the ground with near surface speeds in excess of 50 m/s. The strong wind tends to flow outward radially from where the descending current strikes the earth. The typical size of damaging storms is 6 to 8 km across. At a point beneath the thunderstorm, strong winds may be sustained for up to 30 minutes. In this report, severe thunderstorms are referred to as downbursts.

Mathematical Models

Consider the failure of a transmission line due to thunderstorms is an event A that the wind speed exceeds a prescribed design wind speed and at the same time the thunderstorm strikes the transmission line. The probability of the transmission line failure given that at least one thunderstorm occurs is therefore,

$$P_f = P(A) = P[V > V_D \cap S] \tag{1}$$

where V is the wind speed induced by thunderstorms; V_D is the design wind speed for transmission lines; S is the event that the thunderstorm strikes the transmission line. The probability of transmission line failure under N thunderstorms occurrence is

$$P(A \mid N) = P\left[\bigcup_{i=1}^{N} (V_i > V_D \cap S_i)\right] = P\left[\bigcup_{i=1}^{N} A_i\right] \tag{2}$$

where $P(A \mid N)$ is the conditional probability of A given N; i refers to the ith thunderstorm.

As can be seen, it is difficult in Eq. (2) to determine the probability that a thunderstorm strikes the transmission line. One way to tackle the problem is to make use of geometrical probability, as was done for tornado risk analysis. The geometrical feature of transmission lines is that, in a reference area, only one dimension, i.e., the length of the transmission line is of significance. Accordingly, the geometrical size of a thunderstorm is measured by a path length. The path length of a thunderstorm is defined as the reference distance that the thunderstorm passes with a certain speed. Some data required to estimate the path length may be available from, for example, Dines anemometer charts of meteorological stations. Assume that the ith thunderstorm occurs with a path length b_i. According to the principles of geometrical probability, the probability of a design wind span ℓ of the transmission line being hit is

$$H(b_i) = \begin{cases} b_i \ell / L & \text{if } b_i < L/\ell \\ 1 & \text{if } b_i \geq L/\ell \end{cases} \tag{3}$$

where L is the significant design length of the transmission line in the reference area. Since b_i is a random variable, the probability of a strike by the ith thunderstorm in Eq. (2) is

$$P(S_i) = \int_0^{\infty} P(S_i \mid b_i = b) \cdot f(b) db \tag{4}$$

where f(b) is the probability density function of path length b. Substituting Eq. (3) into Eq (4), it becomes

$$P(S_i) = \int_0^{\infty} H(b) \cdot f(b) db \tag{5}$$

Since the wind speed V and the path length b may be correlated in general, the probability of event A_i in Eq. (2) can be expressed, by substituting Eq. (5), as

$$P(A_i) = P[(V_i > V_D) \cap S_i] = \int_{V_D}^{\infty} \int_0^{\infty} H(b) \cdot f(v, b) dv\, db \tag{6}$$

where f(v, b) is the joint probability density function of V and b. For simplicity, denote Eq. (6) as $Q(V_D)$.

Because the number, N, of thunderstorms that might occur is not known *a priori* in a given time period T, it is appropriate to treat N(T) as a random variable. If it is further assumed that the events that thunderstorms occur are independent of each other, and that the occurrence rate of thunderstorms is constant with time, it may be found that N(T) follows the Poisson distribution. Denoting λ as the mean occurrence rate of thunderstorms, and after some mathematical operations (Li and Holmes, 1994), the final solution of Eq. (1) is

$$P_f = 1 - \exp\left[-\lambda Q(V_D)T\right] \tag{7}$$

where $Q(V_D)$ is obtained from Eq. (6).

Design Wind Speed

For design purposes, Eq. (7) may be further simplified to

$$P_f = \lambda Q(V_D)T \tag{8}$$

obviously, when $\lambda Q(V_D)T$ is small i.e., $\lambda QT < 0.02$, Eqs (7) and (8) are very close. The computation of $Q(V_D)$ i.e., Eq. (6) is quite involved and hence not user-friendly. One way to simplify Eq. (6) is to use conditional probability, provided that conditional distribution is available. By definition of conditional distribution, Eq. (6) can be expressed as:

$$Q(V_D) = \int_{V_D}^{\infty}\int_{0}^{\infty} H(b) \cdot f(v)\, f(b|v)\, dv\, db = \int_{V_D}^{\infty} f(v)dv \int_{0}^{\infty} H(b) \cdot f(b|v)\, db \tag{9}$$

where $f(b|v)$ is the probability density function of path length b, given wind speed $V > V_D$. Substituted Eq. (3) into Eq. (9) and ignoring the almost impossible case of $b_i > L/\ell$, it yields (Li and Holmes, 1994)

$$Q(V_D) = \frac{\ell}{L} G(V_D) \cdot \mu_{b|v} \tag{10}$$

where $G(\ ) = 1 - F(\ )$, i.e. $F(V > V_D)$ and μ denote the mean of the random variable. Eq. (10) is easier to use than Eq. (6). Together with Eq. (8), statistical data that are required to obtain the design wind speed V_D include: (i) Mean occurrence rate λ. (ii) Probability distribution function of wind speed V, F(V). (iii) Conditional mean of path length b given $V > V_D$, $\mu_{b|v}$. When these above statistical parameters are available, the design wind speed can be computed for a given acceptable risk, which is usually expressed in terms of return period R in design practice, i.e.

$$R = \frac{L}{\lambda\, \ell\, G(V_D) \cdot \mu_{b|v}} \tag{11}$$

It is evident that Eq. (11) is very easy to apply.

With typical values of these variables in Table 1, some results of computation are shown in Figures 1 and 2. It can be seen that, for a given design speed V_D, the return period R is very sensitive to distribution parameters, chartacterised by a and u, and vice versa.

Therefore, using accurate probability distribution functions of wind speed V is important in the modelling of severe thunderstorms for transmission line design.

This model will be used with data derived from the improved Dines anemometer records of the Bureau of Meteorology which are currently being analysed.

References

Li, C.Q. and Holmes, J.D. (1994), A Preliminary Study on Models of Severe Thunderstorms for Transmission Line Design, CSIRO Research Report No. 94/82(M).

Standards Association Australia, (1989), AS 1170.2: Loading Code, Part 2: Wind loads, North Sydney, Australia.

Table 1. Typical values of parameters

Parameter	L	ℓ	λ	$\mu_{b\mid v}$
unit	km	km	per year	km
value	150	0.5	1	2

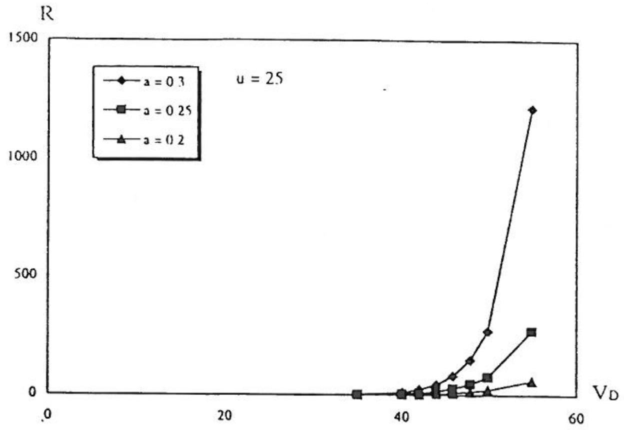

Figure 1. Return period as a function of wind speed (I).

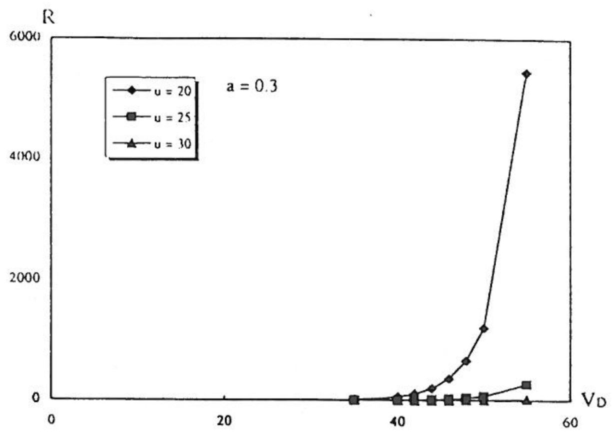

Figure 2. Return period as a function of wind speed (II).