COUPLED RESPONSES OF RECTANGULAR TOWER BUILDINGS UNDER 3-D STOCHASTIC WIND LOAD

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ABSTRACT

This paper presents a method to evaluate acceleration responses of a rectangular tower-like building under normally-incident wind action. The effect of mechanical coupling among along-wind, across-wind and torsion on a multi-degree-of-freedom system is taken into consideration by a mode-superposition approach. The aerodynamic coupling induced by statistic correlations between along-wind, across-wind and torsion load is neglected for those buildings whose side ratio is near unit. This method, based on random vibration theory and wind tunnel test data, can be used to evaluate a tall building based on occupant comfort at the preliminary design stage.

INTRODUCTION

In order to evaluate a tall buildings based on occupant comfort, it is necessary to analyse the dynamic responses of tall buildings by both measurement in wind tunnel tests and theoretical calculations. Because of the complicacy of the problem, existing calculation methods are usually based on some presumptions and premises which can simplify the problem, but unfortunately, are not consistent with the results from wind tunnel tests in some cases. Actually, these calculation methods are only suitable for some special wind attack angles and some special structures with certain geometric shapes and structural characteristics.

On the premise of the data from wind tunnel tests, this paper presents a theoretical method for analysing dynamic responses of the kind of rectangular tall buildings whose side ratios are near unit and structures are asymmetrically arranged under normally incident wind action. In this method, the mechanic model of the tall buildings is a multi-freedom-system whose mass concentrates on every floor of the buildings. The motion of each point on a rigid floor is regarded as the superposition of a translation with mass center and a rotation around the mass center. The mechanical coupling induced by eccentricity between mass and stiffness center, the torsion induced by eccentricity between mass and aerodynamic(geometric) centers are taken into consideration. The aerodynamic coupling induced by statistic correlation between along-wind, across-wind and torsion load are neglected for this kind of tall building.

ASSUMPTIONS AND FORMULAS FOR CALCULATION OF DYNAMIC RESPONSES

Usually, when wind flow is normal to a side face of tall buildings, the total effect of 3-D wind loads on tall buildings reaches its largest value. Therefore only this kind of cases is considered. As it was pointed out by Isyumov(1), and Beneke and Kwok(3), when one side of a tall building is much greater than another and wind flow is normal to its wider face, across-wind load is strongly coupled with torsion load because of the vortex shedding mechanism. In this case, the main part of dynamic torque on tall buildings is induced by wake excitation. In contrast, for square and rectangular tall buildings with close side lengths(especially when wind is acted on their narrow faces), statistic correlation of dynamic wind load between translation and torsion is very small, hence aerodynamic coupled can be neglected. In this case, the dynamic torque induced by wake excitation is also negligible(1), and the asymmetric distribution of dynamic wind pressure induced by turbulent incidence on the four vertical faces of the tall building makes an important contribution to the total dynamic torque. Further investigation shows the dynamic torque on each face of the four is statistic independent from the other(1). Based on the above results from wind tunnel tests, the wind load model for rectangular tall buildings with greater side ratio should be different from that for square and near square tall buildings. When wind acts on the wide face, the wind load on the former should be distinguished from that when wind acts on the narrow face. Since the majority of tall flexible tower buildings belong to the latter, this paper only proposes a wind load model for the latter. The coupled oscillation differential equation for a discrete structure under 3-D wind loads is as follows:

$$\begin{bmatrix} M_{xx} & O & O \\ O & M_{yy} & O \\ O & O & I_{\theta\theta} \end{bmatrix} \cdot \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{pmatrix} + \begin{bmatrix} C_{xx} & O & C_{x\theta} \\ O & C_{yy} & C_{y\theta} \\ C_{\theta x} & C_{\theta y} & C_{\theta\theta} \end{bmatrix} \cdot \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} + \begin{bmatrix} K_{xx} & O & K_{x\theta} \\ O & K_{yy} & K_{y\theta} \\ K_{\theta x} & K_{\theta y} & K_{\theta\theta} \end{bmatrix} \cdot \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} = \begin{pmatrix} P_x \\ P_y \\ P_\theta \end{pmatrix} \quad (1)$$

in which, diagonal matrices M_{xx} ($M_{yy} = M_{xx}$) and $I_{\theta\theta}$ have floor masses m_i and floor moments of inertia I_i as the elements respectively. This equation can be solved by mode-superposition method, and the RMS acceleration of ith mode is as follows

$$\sigma_i = [\int_0^\infty |H_i(i\omega)|^2 \omega^4 S_{F_iF_i}(\omega)d\omega]^{1/2} \quad (2)$$

in which,

$$S_{F_iF_i}(\omega) = = \frac{\Phi_i^T S_{pp}(\omega)\Phi_i}{[\Phi_i^T M\Phi_i][\Phi_i^T M\Phi_i]} \quad (3)$$

is the generalised force spectrum density of ith mode, and

$$\phi_i^T = \{\phi_{x1}, \phi_{x2}, \cdots, \phi_{xk}, \phi_{y1}, \phi_{y2}, \cdots, \phi_{yk}, \phi_{\theta 1}, \phi_{\theta 2}, \cdots, \phi_{\theta k}\} \quad (4)$$

in which,

$$S_{pp}(\omega) = \begin{bmatrix} S_{xx}(\omega) & O & S_{x\theta}(\omega) \\ O & S_{xx}(\omega) & S_{y\theta}(\omega) \\ S_{\theta x}(\omega) & S_{\theta y}(\omega) & S_{\theta\theta}(\omega) \end{bmatrix} \quad (5)$$

The element in $S_{xx}(\omega)$ is the mutual spectrum density of nth and mth floor in along wind direction. The element in $S_{yy}(\omega)$ is the mutual spectrum density of nth and mth floor in across wind direction, in which, the wake excitation spectrum can be expressed by a rational function of the dominant frequency associated with vortex shedding mechanism as in reference (2). The element in $S_{\theta\theta}(\omega)$ is the mutual spectrum density of nth and mth floor in torsion, and it can be expressed as

$$S_{\theta_m\theta_n}(\omega) = S^{[1]}_{\theta_m\theta_n}(\omega) + S^{[2]}_{\theta_m\theta_n}(\omega) \quad (6)$$

in which,

$$S^{[1]}_{\theta_m\theta_n}(\omega) = \rho^2 \bar{v}_l^2 h^2 (\tfrac{z_n}{l})^{\alpha} (\tfrac{z_m}{l})^{\alpha} S_f^{\frac{1}{2}}(z_n,\omega) S_f^{\frac{1}{2}}(z_m,\omega) coh_z(z_n,z_m)[(C_1^2 + C_2^2) \int_{-\frac{B}{2}}^{\frac{B}{2}}\int_{-\frac{B}{2}}^{\frac{B}{2}} y_1 y_2 coh_y(y_1,y_2) dy_1 dy_2 + (C_3^2 + C_4^2) \int_{-\frac{D}{2}}^{\frac{D}{2}}\int_{-\frac{D}{2}}^{\frac{D}{2}} x_1 x_2 coh_x(x_1,x_2) dx_1 dx_2] \quad (7)$$

is the torsion spectrum induced by asymmetric distribution of fluctuation pressure on four vertical sides, in which $S_f(z,\omega)$ is the spectral function of longitudinal fluctuating wind, C_i is the averaged pressure coefficient on ith vertical face.

$$S^{[2]}_{\theta_m\theta_n}(\omega) = d_{y_m} d_{y_n} S_{x_m x_n}(\omega) + d_{x_m} d_{x_n} S_{y_m y_n}(\omega) \quad (8)$$

is the torsion spectrum induced by eccentricity between mass and geometric center, in which, d_y, d_x is the coordinate of mass centre.

$S_{x\theta}(\omega)$, $S_{\theta x}(\omega)$, $S_{y\theta}(\omega)$ and $S_{\theta y}(\omega)$ are mutual spectrum matrix between along-wind direction, across-wind direction and torsion respectively. The element in $S_{\theta x}(\omega)$ and $S_{x\theta}(\omega)$ is

$$S_{\theta_m x_n}(\omega) = S_{x_n \theta_m}(\omega) = d_{y_m} S_{x_m x_n}(\omega) \quad (9)$$

The element in $S_{y\theta}(\omega)$ and $S_{\theta y}(\omega)$ is

$$S_{\theta_m y_n}(\omega) = S_{y_n \theta_m}(\omega) = -d_{x_m} S_{y_m y_n}(\omega) \quad (10)$$

When the contributions of higher modes and mutual variance are neglected, the RMS acceleration of a point (x, y) located on nth floor in x-direction and y-direction can be calculated with following equations respectively

$$\sigma_{\ddot{x}} = [\sum_{i=1}^{3} \sigma_i^2 \phi_{x_{ni}}^2 - 2(y - d_{y_n}) \sum_{i=1}^{3} \sigma_i^2 \phi_{x_{ni}} \phi_{\theta_{ni}} + (y - d_{y_n})^2 \sum_{i=1}^{3} \sigma_i^2 \phi_{\theta_{ni}}^2]^{\frac{1}{2}} \quad (11)$$

$$\sigma_{\ddot{y}} = [\sum_{i=1}^{3} \sigma_i^2 \phi_{y_{ni}}^2 + 2(x - d_{x_n}) \sum_{i=1}^{3} \sigma_i^2 \phi_{y_{ni}} \phi_{\theta_{ni}} + (x - d_{x_n})^2 \sum_{i=1}^{3} \sigma_i^2 \phi_{\theta_{ni}}^2]^{\frac{1}{2}} \quad (12)$$

Therefore, the total acceleration of this point can be express as

$$a_n(x,y) = \mu(\sigma_{\ddot{x}}^2 + \sigma_{\ddot{y}}^2)^{\frac{1}{2}} \quad (13)$$

in which, μ is peak factor.

CALCULATION OF FREQUENCIES AND MODE SHAPES FOR 3-D COUPLED STRUCTURES

Frequencies and mode shapes of 3-D coupled shear type structures can be calculated by free vibration equation which gives the relationship among generalised mass, stiffness matrix and displacement vector. Neglect ing the effect of damping matrix, the equation is given as

$$M\ddot{q} + Kq = O \quad (14)$$

in which K and q are generalised stiffness matrix and generalised displacement vector expressed in Eq.1. In matrix K,

$$[K_{xx}] = \begin{bmatrix} K_{x_1x_1} & K_{x_1x_2} & \cdots & \cdots & O \\ K_{x_2x_1} & K_{x_2x_2} & K_{x_2x_3} & & \\ \vdots & \ddots & \ddots & \ddots & \\ \vdots & & \ddots & \ddots & \ddots \\ O & & & \ddots & \ddots \end{bmatrix} \quad (15); \qquad [K_{\theta\theta}] = \begin{bmatrix} K_{\theta_1\theta_1} & K_{\theta_1\theta_2} & \cdots & \cdots & O \\ K_{\theta_2\theta_1} & K_{\theta_2\theta_2} & K_{\theta_2\theta_3} & & \\ \vdots & \ddots & \ddots & \ddots & \\ \vdots & & \ddots & \ddots & \ddots \\ O & & & \ddots & \ddots \end{bmatrix} \quad (16)$$

$$[K_{x\theta}] = \begin{bmatrix} K_{x_1x_1}e_{y_1} & K_{x_1x_2}e_{y_2} & \cdots & \cdots & O \\ K_{x_2x_1}e_{y_1} & K_{x_2x_2}e_{y_2} & K_{x_2x_3}e_{y_3} & & \\ \vdots & \ddots & \ddots & \ddots & \\ \vdots & & \ddots & \ddots & \ddots \\ O & & & \ddots & \ddots \end{bmatrix} \quad (17); \qquad [K_{y\theta}] = -\begin{bmatrix} K_{y_1y_1}e_{x_1} & K_{y_1y_2}e_{x_2} & \cdots & \cdots & O \\ K_{y_2y_1}e_{x_1} & K_{y_2y_2}e_{x_2} & K_{y_2y_3}e_{x_3} & & \\ \vdots & \ddots & \ddots & \ddots & \\ \vdots & & \ddots & \ddots & \ddots \\ O & & & \ddots & \ddots \end{bmatrix} \quad (18)$$

in which,

$$e_{xi} = d_{xi} - S_{xi} \quad (19)$$

$$e_{yi} = d_{yi} - S_{yi} \quad (20)$$

S_x and S_y are the coordinate of stiffness centre, and

$$K_{\theta_i\theta_j} = K_{\theta_{ij}} + K_{y_iy_j}e_{x_i}e_{x_j} + K_{x_ix_j}e_{y_i}e_{y_j} \quad (21)$$

$k_{\theta ij}$ is the torque at stiffness centre of ith floor induced by unit angular displacement of jth floor.
k_{yy} is just like k_{xx}, and $k_{\theta y}=k_{y\theta}^T$. $k_{\theta x}=k_{x\theta}^T$.

PROPOSAL OF FURTHER INVESTIGATION

Further investigations include the following aspects:
1) The effects of mode-shape coupling induced by eccentricity between elastic and mass centre on acceleration responses;
2) The effects of dynamic torque induced by eccentricity between mass and geometric centre on acceleration responses;
3) The contribution of along-wind, across-wind and torsion to whole acceleration response for corner points of the building;
4) The consistencies between the results of this analysis method and that of wind tunnel tests on aeroelastic models.

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