STUDIES OF NATURAL FREQUENCY AND DAMPING RATIO OF BI-DIRECTIONAL LCVAs

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Abstract

This paper presents an experimental study of geometric properties on the fundamental natural frequency and liquid damping ratio of bi-directional Liquid Column Vibration Absorbers (LCVA) using free-vibration experiments. Natural frequency of liquid oscillation of a LCVA was found to be dependent on its geometric configuration and gravitational acceleration. LCVA liquid damping ratio was found to be dependent on the amplitude of liquid motion in the vertical columns, properties of the LCVA liquid and geometric scale.

Empirical expressions to estimate LCVA natural frequency and liquid damping ratio are proposed. These proposed empirical formulae were obtained from free-vibration experimental results for various square-based bi-directional LCVAs.

1. Introduction

Den Hartog (1947) proposed attaching an auxiliary mass to a main mass to act as a damped passive vibration absorbers. Vibration energy was transferred to the auxiliary mass by tuning its natural frequency to be close to that of the main mass. Energy is then dissipated predominantly by the damping of the auxiliary mass, represented by a viscous damping dashpot. After Watkins (1991) introduced a Liquid Column Vibration Absorber (LCVA), Watkins and Hitchcock (1992) showed that mass ratio, natural frequency and damping ratio of a LCVA are the important parameters to be chosen to successfully mitigate vibrations of a structure. For this reason, Hitchcock et al. (1997) proposed Equation (1) to predict the natural frequency of a bi-directional LCVA in terms of gravitational acceleration (g), and liquid column effective length (L_E).

$$F_a = \frac{1}{2\pi}\sqrt{\frac{2g}{L_E}} \tag{1}$$

Hitchcock et al. (1997) suggested that the value of L_E in Equation (1) could be calculated by Equation (2).

$$L_E = \alpha_{p\eta}\left(\frac{T_{vx}}{T_h}\right)d + 2h_v \tag{2}$$

The empirical factor ($\alpha_{p\eta}$) in Equation (2) was derived for a relatively small sample of square-based bi-directional LCVAs only. Further experiments on various bi-directional LCVA configurations are required to derive a general equation to satisfactorily predict the natural frequency and liquid damping ratio of rectangular-based bi-directional LCVAs

In this paper, empirical formulae are developed from dimensional analysis and multiple regression analysis of free-vibration experimental results to predict the fundamental natural frequency and liquid damping ratio of various bi-directional LCVAs.

2. Dimensional Analysis

Liquid flow in a bi-directional LCVA is dependent on the liquid velocity (v), density of liquid (ρ), dynamic viscosity (μ), gravitational acceleration (g), horizontal liquid column length along x and y axes (d_x and d_y), baffle spacing along x and y axes, (B_{sx} and B_{sy}), vertical column height (h_v), horizontal column thickness (T_h), vertical column thickness along x and y axes (T_{vx}, and T_{vy}), amplitude of liquid oscillation in vertical column (Xo), and orifice opening (T_o). All geometric dimensions of a rectangular-based bi-directional LCVA are presented in Figure 1.

From Buckingham Π Theorem presented in Streeter and Wylie (1981) for example, some functional relationship of the aforementioned quantities must exist, such that
$F(v, \rho, \mu, g, d_x, d_y, B_{sx}, B_{sy}, h_v, T_h, T_{vx}, T_{vy}, Xo, T_o) = 0$
Hence, the functional relationship of dimensionless groupings which represent characteristics of liquid flow in bi-directional LCVA can be presented as,

$$f(\frac{\rho v d_x}{\mu}, \frac{v}{\sqrt{gd_x}}, \frac{d_y}{d_x}, \frac{B_{sx}}{d_x}, \frac{B_{sy}}{d_x}, \frac{h_v}{d_x}, \frac{T_h}{d_x}, \frac{T_{vx}}{d_x}, \frac{T_{vy}}{d_x}, \frac{Xo}{d_x}, \frac{T_o}{d_x}) = 0$$

The first parameter, $\frac{\rho v d_x}{\mu}$, represents Reynolds number $\boldsymbol{R}$, and the second represents Froude number $\boldsymbol{F}$, two of the most important non-dimensional parameters in fluid mechanics.

In the case of a symmetrical square-based bi-directional LCVA without orifices, $d_x=d_y$, $B_s=B_{sx}=B_{sy}$, $T_{vx}=T_{vy}$, and $T_h=T_o$. Characteristics of liquid flow in the LCVA may then be written as,

$$f(\frac{\rho v d_x}{\mu}, \frac{v}{\sqrt{gd_x}}, \frac{B_s}{d_x}, \frac{h_v}{d_x}, \frac{T_h}{d_x}, \frac{T_{vx}}{d_x}, \frac{Xo}{d_x}) = 0 \quad (3)$$

3. Study of Bi-directional LCVA Natural Frequency

The fundamental natural frequency of a square-based bi-directional LCVA can then be related to liquid flow by considering Equations (1) and (3). The value of L_E, presented in Equation (1), can be non-dimensionalised with respect to horizontal column length (d_x), and $\frac{L_E}{d_x}$ can be written as Equation (4).

$$\frac{L_E}{d_x} = f(\frac{\rho v d_x}{\mu}, \frac{v}{\sqrt{gd_x}}, \frac{B_s}{d_x}, \frac{h_v}{d_x}, \frac{T_h}{d_x}, \frac{T_{vx}}{d_x}, \frac{Xo}{d_x}) \quad (4)$$

Free-vibration experiments were performed to determine the natural frequencies and liquid damping ratios of different bi-directional LCVA configurations filled with fresh water. These LCVA configurations represented the following variations in the dimensionless parameters listed in Equation (4): $\frac{h_v}{d_x}$ was varied from 0.078 to 1.298; $\frac{T_h}{d_x}$ was varied from 0.022 to 0.127; $\frac{T_{vx}}{d_x}$ was varied from 0.049 to 0.140. The value of $\frac{B_s}{d_x}$ was varied from 0.122 to 0.994, where a value of 0.122 corresponds to the configuration in which baffles divided the vertical column into eight, almost equal, portions.

From a multiple regression analysis of the experimental results for significant parameters, an empirical expression for the prediction of $\frac{L_E}{d_x}$ values was developed and is presented in Equation (5).

$$\frac{L_E}{d_x} = -0.22\left(\frac{B_s}{d_x}\right) + 1.38\left(\frac{h_v}{d_x}\right) - 18.21\left(\frac{T_h}{d_x}\right) + 13.05\left(\frac{T_{vx}}{d_x}\right) + 1.73 \quad (5)$$

It can be seen that Equation (5) is governed by a constant 1.73. In addition, the parameters of $\frac{T_h}{d_x}$ and $\frac{T_{vx}}{d_x}$ have the potential to be replaced by $\frac{T_{vx}}{T_h}$, which is representative of the ratio of vertical to horizontal column cross-sectional area. By eliminating the constant and combining the aforementioned parameters, Equation (5) can be rewritten as Equation (6).

$$\frac{L_E}{d_x} = -0.25\left(\frac{B_s}{d_x}\right) + 1.35\left(\left(\frac{h_v}{d_x}\right) + \left(\frac{T_{vx}}{T_h}\right)^{\frac{2}{3}}\right) \tag{6}$$

A comparison between LCVA natural frequencies determined from the experiments and those predicted using Equations (1) and (6) is presented in Figure 2. In Figure 2, the discrepancies between predicted and measured natural frequencies are generally within $\pm 5\%$. It is expected that natural frequency mistuning of $\pm 5\%$ or less can be compensated for by fine-tuning LCVA natural frequency after installation. Experimental data lying outside the range of $\pm 5\%$ correspond to very small scale devices which have limited practical use.

4. Study of Bi-directional LCVA Damping Ratio

The liquid damping ratio (ξ) of a bi-directional LCVA can be calculated from the free-vibration experiments from the logarithmic decrement of the peak liquid oscillation over n cycles, as presented in Equation (7).

$$\xi = \frac{1}{2\pi n} \ln \frac{Xo_i}{Xo_{i+n}} \tag{7}$$

where Xo_i and Xo_{i+n} are the amplitude of liquid oscillation in the vertical columns at cycle i and cycle $i+n$ respectively.

From the dimensional analysis in Section 2, the LCVA liquid damping ratio (ξ) can be presented in Equation (8) as a function of the dimensionless parameters as presented in Equation (3). In Equation (8), ξ is expressed as a percentage of critical damping.

$$\xi = f\left(\frac{\rho v d_x}{\mu}, \frac{v}{\sqrt{g d_x}}, \frac{B_s}{d_x}, \frac{h_v}{d_x}, \frac{T_h}{d_x}, \frac{T_{vx}}{d_x}, \frac{Xo}{d_x}\right) \tag{8}$$

From a multiple regression analysis of the free-vibration experimental results, an empirical formula for the estimation of ξ was developed. By again eliminating the constant and combining $\frac{T_h}{d_x}$ and $\frac{T_{vx}}{d_x}$ into the more meaningful dimensionless parameter $\frac{T_{vx}}{T_h}$, which is representative of the ratio of vertical to horizontal column cross-sectional area, as presented in Equation (9)

$$\xi = -4.0x10^{-5}\left(\frac{\rho v d_x}{\mu}\right) - 580\left(\frac{v}{\sqrt{g d_x}}\right) - 3.2\left(\frac{B_s}{d_x}\right) + 8.0\left(\frac{h_v}{d_x}\right) + 1.0\left(\frac{T_{vx}}{T_h}\right) + 300\left(\frac{Xo_i}{d_x}\right) \tag{9}$$

The experimentally determined liquid damping ratios of all tested bi-directional LCVA configurations were plotted with respect to liquid damping ratios predicted using Equation (9), as presented in Figure 3. It can be seen from Figure 3 that discrepancies between predicted and measured liquid damping ratios may be relatively large, although this is not attributable to any one factor. LCVA liquid damping ratio was found to experience significant variability due to many factors controlling it. However, of the three parameters which control the effectiveness of a LCVA, namely mass ratio, tuning ratio and liquid damping ratio, liquid damping ratio is the least critical parameter, provided that the other two parameters are close to optimum. The empirical formula proposed for LCVA liquid damping ratio will provide a reasonable estimate for the purposes of LCVA design, particularly for larger scale LCVAs, which are more practical, and at larger amplitudes of LCVA liquid oscillation, when dissipation of vibrational energy is required most.

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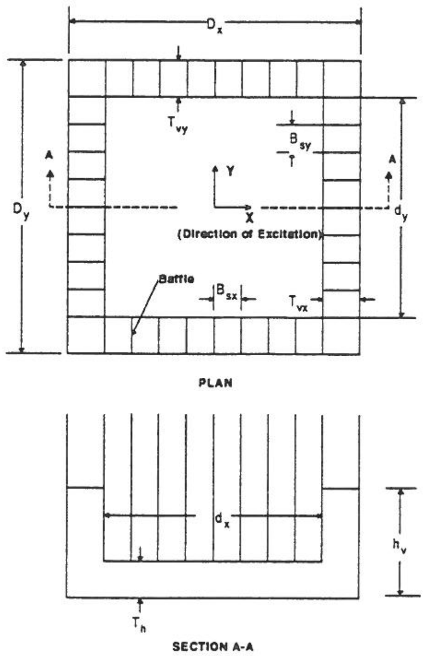

Figure 1. Bi-directional Liquid Column Vibration Absorber (LCVA)

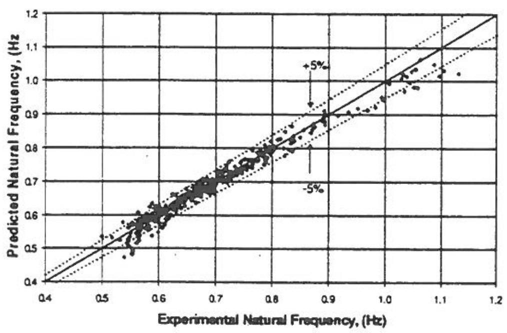

Figure 2. Comparison of Experimental and Predicted Natural Frequencies of Bi-directional LCVAs

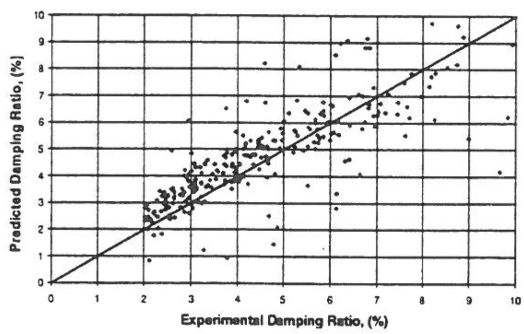

Figure 3. Comparison of Experimental and Predicted Damping Ratios of Bi-directional LCVAs