DISTRIBUTIONS OF EXTREME PRESSURE COEFFICIENTS

J.D. Holmes (Monash University and JDH Consulting)
L.S. Cochran (CPP Inc.)

Introduction

In previous studies, authors have fitted an extreme value distribution to the largest pressure coefficient measured in repeated wind tunnel samples equivalent to time periods of 10 to 60 minutes in full scale. In almost all cases, the two-parameter Type I extreme value distribution has been fitted to the data. Reasonable fits have been obtained, although relatively few samples (usually less than 100) have been used. The Type I Extreme Value distribution is simple to fit, but has the disadvantage that it is unbounded when projected out to low risks of exceedence, a result which does not seem intuitively correct for real physical phenomena. Some recent analyses of extreme wind speeds have investigated the use of the three-parameter Generalised Extreme Value (G.E.V.) distribution (e.g. Holmes and Moriarty, 1999).

It was shown by Jenkinson (1955) that the Type I E.V. distribution is a special case of the G.E.V. The cumulative probability distribution of the G.E.V. is given by Equation (1).

$$F_x(x) = \exp\{-[1 - k(x-u)/a]^{1/k}\} \tag{1}$$

In Equation (1), u is a location parameter
a is a scale parameter
k is a shape factor

As k tends to 0, Equation (1) becomes Equation (2) in the limit. Equation (2) is the *Type I Extreme Value Distribution*, or *Gumbel* Distribution.

$$F_x(x) = \exp\{-\exp[-(x-u)/a]\} \tag{2}$$

A positive value of shape factor (Type III E.V. distribution) is of particular significance, as the distribution in this case will predict a theoretical upper limit to the variate, x. This upper limit is given by Equation (3).

$$x_{max} = u + \left[\frac{a}{k}\right] \tag{3}$$

In the present paper, several thousand repeated samples of extreme pressure coefficients from two measurement positions on a low-rise building model have been used to better determine the appropriate probability distributions.

Measurements

Measurements were obtained from a 1/100 scale model of the Texas Tech Building, using the Meteorological Wind Tunnel at Colorado State University. Measurements were carried out in a simulated boundary flow in rural terrain (RII). This flow represented a boundary layer with a full-scale roughness length of 10 mm. The longitudinal turbulence intensity at the top of the building (4 metres in full scale) was about 0.25, and the integral turbulence turbulence scale was 67 metres. The frequency response of the pressure measurement system was near flat in amplitude to a full-scale frequency of about 2.5 Hertz. Each data sample from which the maximum or minimum pressure coefficient was extracted was 18 second in length and represented about 30 minutes in full scale time. Full details of the experimental methods used are given by Cochran (1992).

Measurements were taken from roof tapping 50501 near the corner of the roof of the building for a wind direction of 215 degrees, and from wall pressure tapping 42206, near the centre of the west wall, for a wind direction of 270 degrees. 5100 samples of minimum pressure coefficients were obtained for the roof pressures (25.5 hours of wind-tunnel time), and 4200 samples of maximum pressures were obtained for the wall pressures.

Analysis

The data was first fitted with a Type I distribution. The data were ordered and allocated a Gringorten plotting position, given by :

$$p = (i-0.44)/ (N+1-0.88)$$

where i is the order, and N is the total number of values. This plotting parameter is nearly unbiased for this distribution, unlike the Gumbel plotting position (i/(N+1)) (Gringorten, 1963). The plotting parameter was transformed into a reduced variate $-\log_e(-\log_e(1-p))$, and the recorded Cp's were plotted against the reduced variate.

Figures 1, 2 show the fitting of the Type I for the 5100 minimum roof pressures and 4200 maximum wall Cp's, respectively. The fitting is reasonable in the case of the wall pressures, but the roof pressures (Figure 2) depart considerably from the fitted line at the right hand end.

The data was also fitted with the Generalised Extreme Value Distribution, using the method of probability weighted moments (Hosking, Wallis and Wood, 1985). In this method a plotting parameter is formed as follows :

$$p = (i-0.35)/N$$

The probability weighted moments of 0th, 1st and 2nd orders are estimated by multiplying the recorded data points by p^0, p^1 and p^2 respectively and averaging over the data set. These estimates are compared with the theoretical values of these moments to estimate the parameters, a,u and k.

The results of this fitting are given in Figures 3 and 4. The parameters obtained are listed in Table I. There is a clear curvature in the roof pressures, corresponding to a positive shape factor, k. The fit is less good for the wall pressures, with a lower value of k, indicating lower curvature. However, a non-zero shape factor indicates that a Type III is a better fit than the Type I E.V. distribution.

Table I. parameters in G.E.V. fit to extreme roof and wall pressures

	a	u	k	Upper limit
Roof pressures	-0.52	-5.42	0.086	-11.52
Wall pressures	0.17	1.73	0.070	4.11

$M_0 = \frac{1}{N}\sum p_i^0 C_{pi}$ $M_1 = \frac{1}{N}\sum p_i^1 C_{p,i}$ $M_2 = \frac{1}{N}\sum p_i^2 C_{pi}$

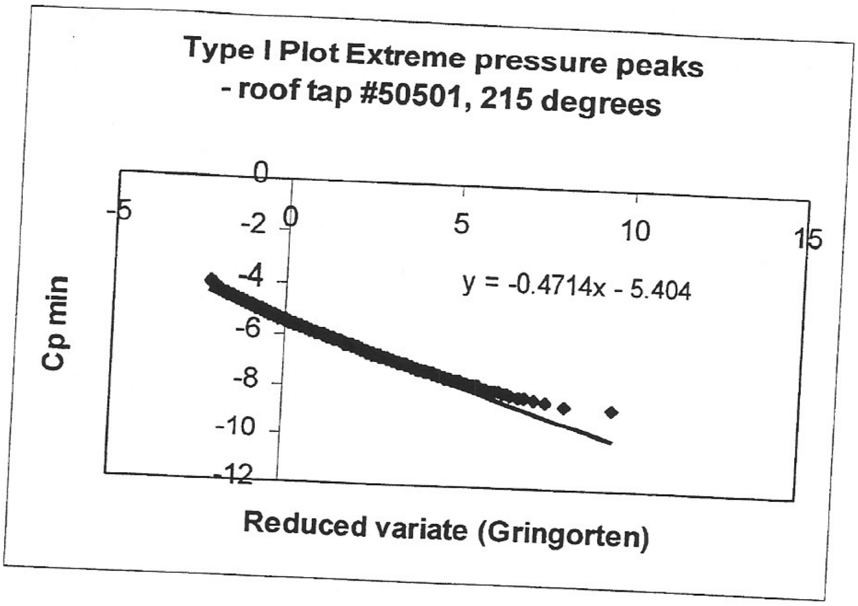

Figure 1. Type I extreme value distribution fit to roof minimum pressure coefficients

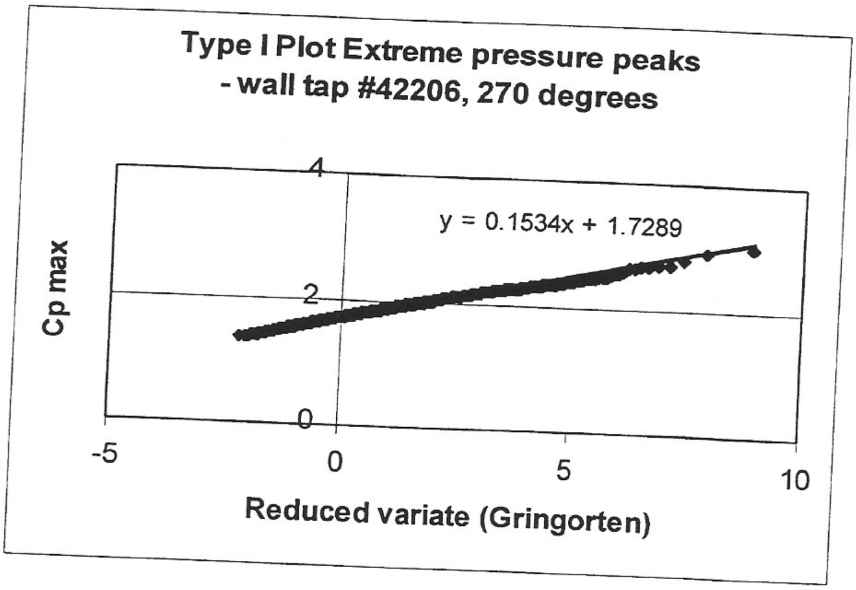

Figure 2. Type I extreme value distribution fit to wall maximum pressure coefficients

Comments and Conclusions

The G.E.V. fit illustrated in Figures 3 and 4, and with the parameters tabulated in Table I, indicate that both the roof and wall pressure coefficients are best fitted with a distribution with a positive shape factor. This indicates theoretical upper limits to the extreme pressures of -11.5 and +4.1, respectively. Such behaviour is to be expected as clearly a wind tunnel will not be able to generate pressures of unlimited magnitude. These upper limits are significantly beyond the range of the data, and to be better defined, even more data points are required.

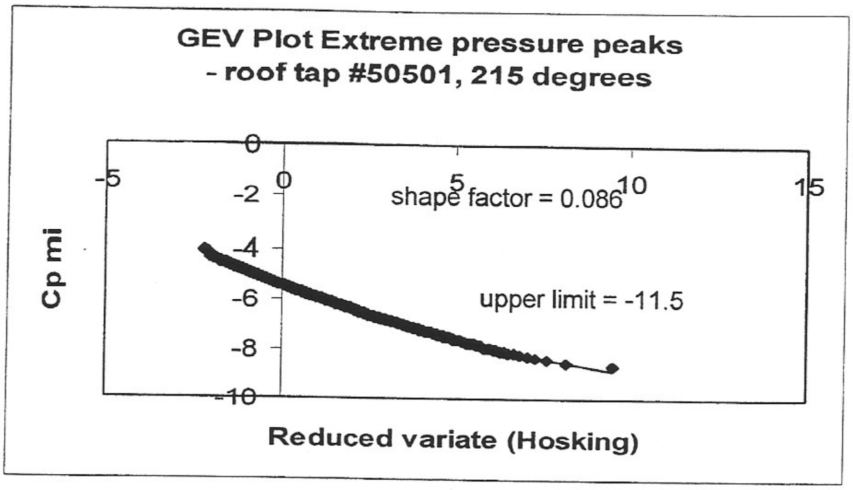

Figure 3. Generalised extreme value distribution fit to roof minimum pressure coefficients

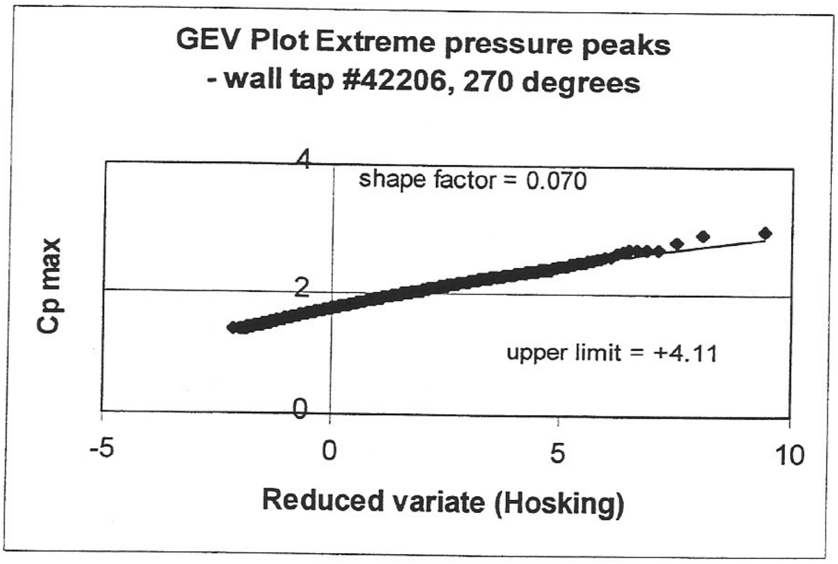

Figure 4. Generalised extreme value distribution fit to wall maximum pressure coefficients

References

Cochran, L.S. Wind-tunnel modelling of low-rise structures. Ph. D. Thesis. Colorado State University, 1992.

Gringorten, I.I. A plotting rule for extreme probability paper. *Journal of Geophysical Research*, Vol. 68, pp 813-814, 1963.

Holmes, J.D.and Moriarty, W.W. Application of the Generalized Pareto distribution to extreme value analysis in wind engineering. *Journal of Wind Engineering and Industrial Aerodynamics*, Vol. 83, pp1-10, 1999.

Hosking, J.R.M., Wallis, J.R., and Wood, E.F. Estimation of the Generalized Extreme Value Distribution by the method of probability-weighted moments. *Technometrics,* Vol. 27, pp251-261, 1985.

Jenkinson, A.F. The frequency distribution of the annual maximum (or minimum) values of meteorological elements. *Quarterly Journal of the Royal Meteorological Society*, Vol. 81, pp158-171, 1955.