WIND ACTION ON GLASS AND BROWN'S INTEGRAL

J D Holmes*

Introduction

The action of wind pressure is a major design consideration in the design of glass and its fixing in buildings of all types. It is well known that wind pressures on structures fluctuate highly with time, and also that the strength of glass is highly dependent on the duration of the loading. The interaction of these two phenomena produces a design problem of great complexity. There have been many failures of glass cladding in the last ten years, particularly in North America, and this has resulted in much litigation. However, the failures have also resulted in extensive research which has improved our understanding of the problem.

This paper examines, in a general way, the action of fluctuating wind pressure on glass, making use of the 'Brown's Integral' formulation of glass damage, and a probability distribution model of wind pressures on buildings previously suggested by the author [1].

Brown's Integral

The surfaces of glass panels are covered with tiny flaws of various sizes and orientations. When these flaws are exposed to tensile stresses, they grow at a rate dependent on the magnitude of the stress field (as well as relative humidity and temperature). The result is a strength reduction which is dependent on the tensile stress and the duration of the stress.

Drawing on earlier studies of this phenomenon, which is known as 'static fatigue', Brown [2] suggested a formula for damage accumulation which, at constant humidity and temperature, has the form:

$$D = \int_o^T [\sigma\ (t)]^n\ dt \qquad (1)$$

where T = the time over which the glass is stressed; $\sigma(t)$ = the time varying stress; and n = a high power.

Although this formulation has disadvantages, for example it implies that cracks grow even at very low levels of stress, an assumption that contradicts the ideas of fracture mechanics, it is able to explain many of the observed characteristics of glass failure, and has been incorporated into several design models of glass strength, e.g. Minor [3], Beason and Morgan [4].

If it is assumed that the stress in the vicinity of the critical flaw is proportional to the applied loading on the glass pane, i.e. the wind pressure, then equation (1) can be written:

$$D = k \int_o^T [p\ (t)]^n\ dt$$

$$= kT.(\tfrac{1}{2}\rho\bar{u}^2)^n \int_o^\infty C_p^n\ .\ f_{C_p}\ (C_p)\ dC_p \qquad (2)$$

---

* CSIRO Division of Building Research, Melbourne, Australia.

where k = a constant; ρ = the density of air; $\bar{u}$ = a reference mean wind speed during a windstorm; $C_p(t)$ = the time varying pressure coefficient; and $f_{c_p}(C_p)$ = the probability density function (pdf) for C_p.

Thus, from equation (2), the damage integral appears as the nth moment of the probability density function of the pressure coefficient. A value of n of 16 has been suggested as appropriate [3,4].

Probability Distributions for Wind Pressures

The evaluation of equation (2) requires knowledge of the probability density function for wind pressure fluctuations on buildings. The equation has been evaluated for a Gaussian function by Allen and Dalgleish [5]. However, the Gaussian model, although a reasonable model for wind velocity fluctuations, is not adequate for wind pressures, which are highly skewed. Mayne and Walker [6] carried out computations with a distribution which was bilinear on Gaussian probability paper, with the Gaussian form in the range of two standard deviations from the mean.

A more realistic model of the probability distribution of wind pressures is that described by Holmes [1]. This is based on a Gaussian model of the wind speed upwind of the building. The nonlinear relationship between pressure and wind speed produces a skewed distribution for wind pressure, of the correct form.

If a quasi-steady relationship between the surface pressure fluctuations and the upwind velocity fluctuations is assumed, then we can write:

$$p(t) = C_{po} . \tfrac{1}{2}\rho . [u(t)]^2$$

where C_{po} is a constant, quasi-steady pressure coefficient. Then the fluctuating pressure coefficient is:

$$C_p(t) = C_{po} \frac{[u(t)]^2}{\bar{u}^2}$$

Neglecting the contribution from negative values of u(t), the pdf of $C_p(t)$ can be written [1]:

$$f_{c_p}(C_p) = \frac{1}{\sqrt{2\pi\, C_p/C_{po}}\; 2I_u . C_{po}} \left\{\exp\left[-\frac{1}{2}\left(\frac{\sqrt{C_p/C_{po}} - 1}{I_u}\right)^2\right]\right\} \qquad (3)$$

where I_u is the turbulence intensity, $\sigma_u/\bar{u}$.

Equation (3) is a good model for windward wall pressures (C_{po} positive) as is shown in [1]. For separated flow regions (C_{po} negative), equation (3) is not such a good model, but the general form and the skewness is approximately correct. It can be used as an adequate model to investigate the damage accumulation of glass under wind. Equation (3) is shown in Figures 1 and 2 for various values of I_u; the increasing skewness of the distribution as I_u increases is clearly shown.

Computation of Damage Due to Wind Action

The integral in equation (2) is proportional to the rate at which damage is accumulated in the glass panel. The integral has been evaluated for the probability distribution given by equation (3) for various values of I_u. The results are shown in Table I.

TABLE I

I_u	Relative damage accumulation rate
0.10	5.06×10^{1}
0.15	1.46×10^{3}
0.20	4.44×10^{4}
0.25	1.17×10^{6}
0.30	2.53×10^{7}

The contributions to the damage integral from various ranges of pressure coefficient are shown as a normalized function, $g_{C_p}(C_p)$, in Figures 1 and 2.

Discussion

Table I shows that the rate of damage accumulation increases rapidly as the turbulence level associated with the pressure fluctuations increases. The r.m.s. pressure coefficient is related to the turbulence intensity by the following equation [1]:

$$\sigma_{C_p} = C_{p_o} . 2I_u \left[1 + \frac{1}{2} I_u^2\right]^{\frac{1}{2}} \qquad (4)$$

Figures 1 and 2 show that the contributions to the damage rate are spread over a fairly wide range of the pressure fluctuations. Also the range increases with increasing I_u, σ_{C_p} and skewness of the distribution. However, consideration of the frequency content and cycling rate of the pressure fluctuations for wind loading shows that a large part of the contribution to the integral, including the peak region of the curves, comes from isolated peaks which occur at infrequent intervals, perhaps only one or two during a given wind storm associated with the mean wind speed, u. This adds support to the assumption in design of a single peak load from wind tunnel tests or from a wind loading code. However, it also supports the need for accurate measurement of these pressure peaks. The high weighting given to the pressure coefficient in the integral of equation (2) ensures that any attenuation of the pressure peaks due to unsatisfactory frequency response of the measuring system, for example, will significantly underestimate the computed damage rate. These aspects are clearly worthy of further investigation.

Conclusions

Assuming that Brown's Integral is a valid representation of the damage rate experienced by a glass panel under fluctuating loading, this study has shown that the rate of accumulation of damage, under wind loading, is strongly affected by the level of pressure fluctuations and the skewness of the probability distribution of the wind pressures.

It appears that most of the damage is caused by large pressure or suction peaks which occur at infrequent intervals due to the intermittent nature of wind pressures. The ramifications of this for simple design methods, and for the frequency response of pressure measuring instrumentation in wind tunnel tests, are significant.

References

1. J.D. Holmes, 'Non-Gaussian Characteristics of Wind Pressure Fluctuations', J. Wind Engg & Ind. Aerodyn., 7, pp.103-108, 1981.

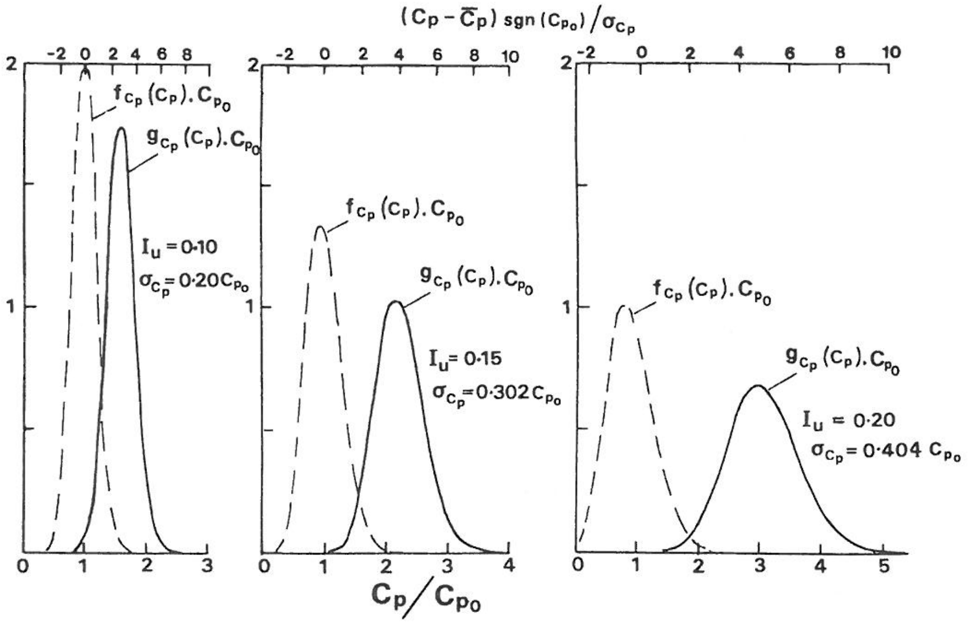
2. W.G. Brown, 'A Load Duration Theory for Glass Design', Div. Build. Res., NRC of Canada, Research Paper No.508, 1972.

3. J.E. Minor, 'Window Glass Design Practices : A Review', J. Struct. Engg, 107, pp.1-12, 1981.

4. W.L. Beason and J.R. Morgan, 'Glass Failure Prediction Model', J.Struct. Engg, 110, pp.197-212, 1984.

5. D.E. Allen and W.A Dalgleish, 'Dynamic Wind Loads and Cladding Design', Div. Build. Res., NRC of Canada, Research Paper No.611, 1973.

6. J.R. Mayne and G.R. Walker, 'The Response of Glazing to Wind Pressure', Building Research Establishment (UK), Current Paper CP44/76, 1976.



$$g_{C_p}(C_p) = C_p^{16} \cdot f_{C_p}(C_p) \Big/ \int_0^{\infty} C_p^{16} \cdot f_{C_p}(C_p)\, dC_p$$

Figure 1 Pdf of pressure fluctuations and contribution to damage integral

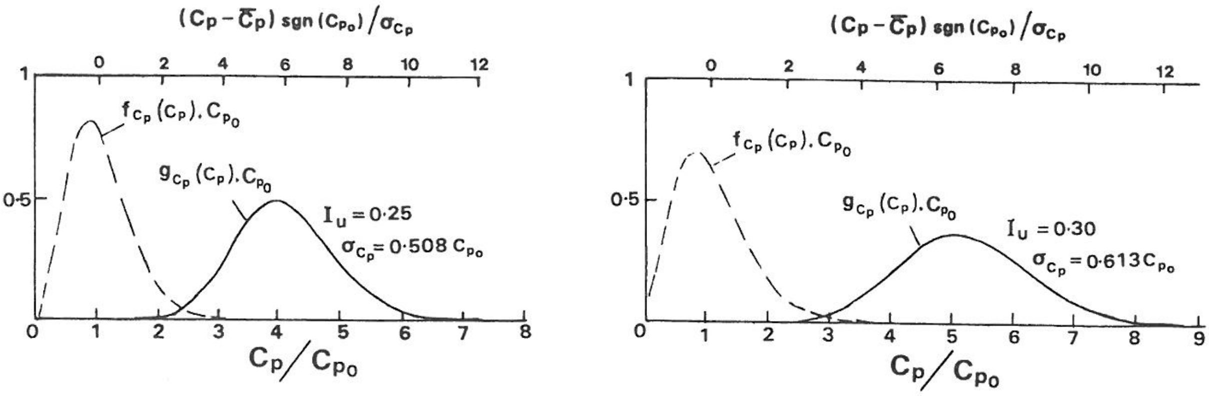


Figure 2 Pdf of pressure fluctuations and contribution to damage integral