CUMULATIVE DAMAGE FROM WIND LOADS ON GLASS

S G Reid*

Introduction

An important aspect of wind engineering is the design of glass cladding to resist wind loads. The choice of the appropriate wind load for the design of glass cladding depends on an interaction between the anticipated wind loads and the glass strength. A widely-accepted model of this interaction is given by Brown's Load-Duration Theory of Glass Strength [2], which is the basis of an advanced probabilistic model of glass strength under general load conditions [1].

Brown's Theory is a variation of Weibull's Statistical Theory of the Strength of Materials [5], incorporating the glass 'fatigue' failure criterion of Charles [3,4] to account for load-duration effects. Weibull's Theory is based on the assumption that the probability distribution function of the strength S of a brittle body can be expressed in terms of 2 strength parameters k and m, such that

$$F_S(s) = 1 - \exp[-ks^m]$$

where ks^m is the expected number of cracks that would fail at a stress less than or equal to s. Charles' criterion for fatigue failure is

$$\int_0^{t_f} s^n \, dt = \text{constant}$$

where t_f denotes the time to failure. Brown used Charles' failure criterion to relate general load conditions to equivalent loads of standard form, to which Weibull's strength theory was applied.

This paper examines the validity of Brown's Theory from a fundamental theoretical point of view. Some limitations of the theory are noted, and recommendations are given concerning its application in the design of glass to resist wind loads.

2. The Time-Dependence of Glass Strength

Qualitative results for the time-dependence of glass strength can be obtained from a simple model consistent with atomice lattic models for atomically sharp brittle cracks propagating by the sequential rupture of bonds, with crack velocities according to the theory of thermal fluctuations and the principles of stochastic mechanics. Thus, a crack will propagate when the crack tip stress σ exceeds a certain value σ_p, and a crack will fail when the crack tip stress reaches a critical value σ_c.

2.1 The time dependent distribution of crack strength

The resultant effect of stress-histories on the distribution of crack strengths is complex, but it can be simply demonstrated in a qualitative sense, for the particular case that the stress history comprises a sustained nominal stress s_i of duration T_i denoted $\{s_i, T_i\}$. In that case, a smooth distribution of initial crack strengths would be transformed as indicated in Figure 1, with regard to the probability density function of crack strengths $f_S(s)$ (with the probability mass of failed cracks concentrated in an atom at $s \cong 0$).

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*CSIRO Division of Building Research, Melbourne.

Furthermore, it can be shown similarly that the for 2 sequential stress regimes $\{s_i, T_i\}$ and $\{s_j, T_j\}$, the combined effect on the distribution of crack strengths depends on the ordering of the regimes. Thus the effects of sequential stress regimes are not independent.

2.2 The time-dependent probability of crack failure

The probability of failure of a crack during a stress regime is often of interest. This corresponds to the probability mass of failed cracks concentrated in an atom of s = 0 due to the transformation of the distribution of crack strengths. In general, this probability is a complex function of the stress history, but for a stress regime {s,T} it can be expressed in terms of a joint probability distribution function $F_{S,T}(s,t)$, denoting the probability that a crack will fail at a nominal stress s or less, in a time t or less. Clearly

$$F_{S,T}(s,o) \leq F_{S,T}(s,t) \leq F_{ST}(s,\infty)$$

Furthermore

$$F_{S,T}(s,o) = F_{S_o}(s)$$

and

$$F_{S,T}(s,\infty) = F_{P_o}(s) = F_{S_o}(s\ \sigma_c/\sigma_p)$$

where s_o is the initial crack strength and P_o is the initial stress required to propagate the crack. Thus, the bounds on $F_{S,T}(s,t)$ are of the form of $F_{S_o}(s)$.

However, for finite t it can be shown that $F_{S,T}(s,t)$ is not of that form, i.e.

$$F_{S,T}(s,t) \neq F_{S,T}(\alpha_t s) \quad ; \quad 0 < t < \infty$$

where α_t is a function of t alone. The bounds of $F_{S,T}(s,t)$ and the form of the function for finite t are indicated in Figure 2.

2.3 Equivalence of stress histories

Various stress histories would be strictly equivalent in their effects on glass strength only if their effect on FS(s) were identical. However, a limited equivalence of stress histories can be established on the basis of identical probabilities of failure during the various stress histories. In terms of probability distributions of crack strengths, this yields an equivalence of probability masses concentrated at s = 0, rather than an equivalence of complete probability distributions.

Conditions for limited equivalence are those for propagating a certain limiting crack (and all weaker cracks) to failure. General conditions for limited equivalence are complex and unknown. Approximate conditions for limited equivalence have been obtained empirically for standard load patterns, but such approximations are of limited validity.

3. Review of Brown's Load Duration Theory of Glass Strength

Reviewing Brown's Theory in the light of the above results, reveals flaws in Brown's formulation. An obvious inconsistency concerns the stress at which cracks propagate. Brown's Theory does not model the non-propagation of small cracks (as shown in Figure 1). Similarly, Brown's Theory implies that the effects of sequential stress regimes are independent, which is not realistic.

Another inconsistency concerns the glass strength parameter m. Brown's Theory implies that m is independent of the stress history, but this is inconsistent with the proposed model, according to which the glass strength parameter m would be a complex function of the stress history (reflected in the form of $F_{S,T}(s,t)$).

4. Conclusions and Recommendations

Brown's Theory is inconsistent with a realistic model of the time-dependence of glass strength, so it is not a reliable basis for relating the glass strength under standard load tests to the glass strength under wind loads. To minimize errors in the application of Brown's Theory, glass strength tests should simulate the failure conditions under wind loads.

References

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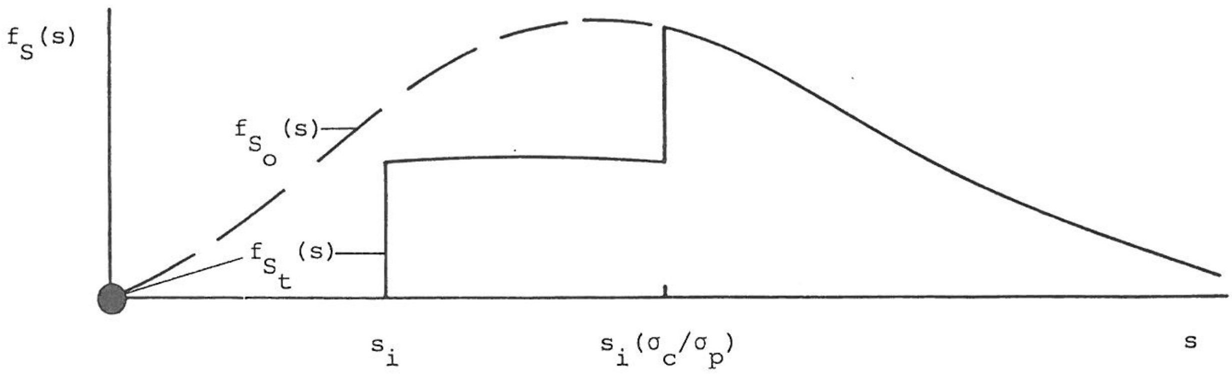

Figure 1 The time-dependent probability distribution of crack strength

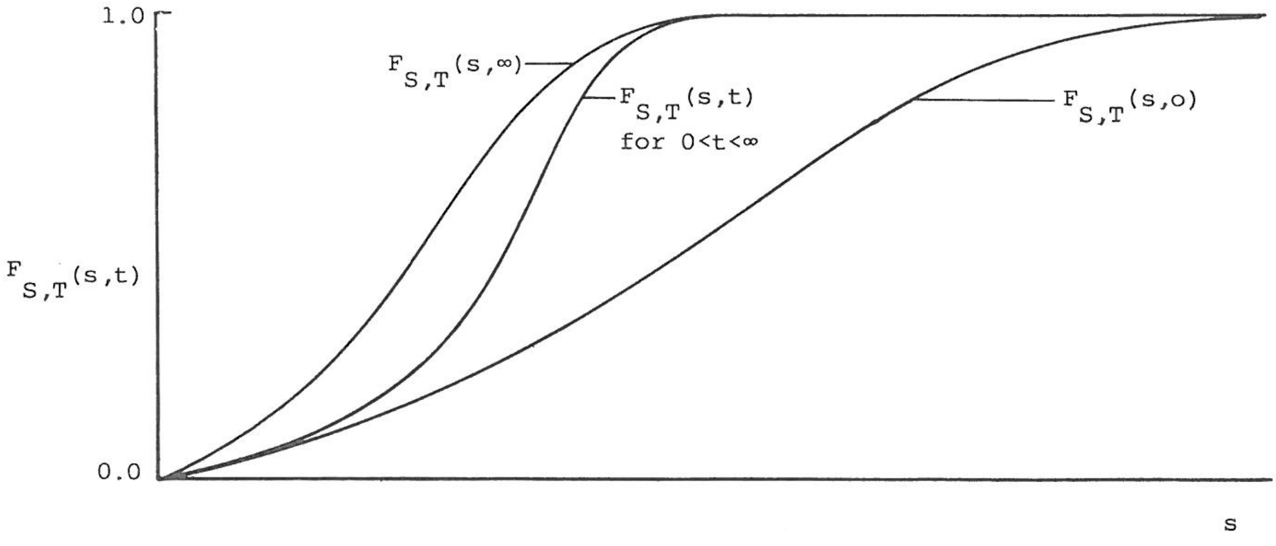

Figure 2 The time-dependent probability of crack failure