PRESSURE FLUCTUATION TRANSMISSION IN SMALL DIAMETER TUBES - COMPARISON OF THEORY AND EXPERIMENT

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Introduction

Knowledge of the transmission characteristics for pressure fluctuations of small diameter tubes is important for many practical measurement situations in wind engineering, aerodynamics and aeroacoustics. Although the theoretical problem of propagation of sound waves in gases in cylindrical tubing is a classical one, associated with such famous names as Helmholtz, Kirchhoff and Rayleigh, most of the early approximate solutions (for low reduced frequencies) have been superseded by those developed by Iberall [1] and Bergh and Tijdeman [2]. A review of the many existing analytical solutions has been given by Tijdeman [3].

Recent experimental developments have been the introduction of very small diameter restrictor tubes [4], and pneumatic averagers involving parallel tube systems, [5]. Gumley [6, 7] extended the Bergh and Tijdeman theory to include the pneumatic averagers, and advocated the use of the theory for designing tubing systems in practical situations.

The interest of the Division of Building Research in the problem is essentially a practical one - the optimization of tubing systems for the measurement of fluctuating and peak pressures and area loads on wind tunnel models of buildings in simulated atmospheric boundary layers. The frequency response requirements for accurate measurements of peak pressures in these situations have been defined [8], and an experimental rig has been set up to carry out dynamic calibration of complete pressure measurement systems, including pressure transducers and associated volumes [9]. The present paper makes some comparisons between system responses measured with the calibration rig, and predicted by the theory, as formulated by Bergh and Tijdeman, and Gumley.

Assumptions of the Theory

The derivation of the theory is fairly lengthy, but described in detail in References [2] and [6]. The motion of the fluid in a tube of circular cross-section is described by the fundamental flow equations: the Navier-Stokes equations of momentum conservation, the equation of continuity, the equation of state, and the energy equation. The following assumptions are made:

(i) the sinusoidal pertubations in pressure, density, temperature and velocity are small in comparison to the mean values.

(ii) The length to diameter ratios of the tube sections are assumed large so that end effects are negligible.

(iii) The Reynolds Numbers are low enough so that the flow is laminar throughout the system.

(iv) The thermal conductivity of the wall of the tubes is assumed to be large, so that temperature fluctuations at the wall are zero.

(v) The material of the tube walls is assumed to be rigid.

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(vi) The cross flow velocity at the entrance to the tubes is assumed small.

(vii) When the averaging manifold is included, it is assumed to have rigid walls, and that there are no spatial variations of fluid properties within the volume.

(viii) The pressure expansions in the tubing, manifold and transducer volume are assumed to be polytropic processes.

With these assumptions, Bergh and Tijdeman [2] derived a recursion equation for the complex ratio of pressure amplitudes across an element consisting of a tube and a volume. With the assumptions made, the equation is valid for values of reduced frequency $\omega D/a_0$ much less then unity, where ω is the circular frequency of pressure fluctations, D is the tube diameter and a_0 is the speed of sound. This condition is normally easily satisfied for most practical measurement conditions and frequency requirements. Gumley [6] derived a similar equation for an element consisting of m identical parallel tubes feeding a single volume. In this case, the equation gives the ratio of the *average* of the pressure amplitudes at the inlets to the m tubes, and the pressure amplitude in the volume.

The Bergh and Tijdeman element can be used to represent a series of tubes of different diameters, including restrictors, by setting all volumes except the last (adjacent to the transducer) equal to zero. The Gumley element can be used to represent a pneumatic averager (manifold) system at the input end of a series tube system [6]. For the present results, the theoretical equations were programmed in BASIC, and a microcomputer used to compute the response characteristics.

Single Tube Comparisons

Figure 1 shows the computed amplitude ratio and phase response for a simple constant diameter tube of 1.5 mm internal diameter and 500 mm length connected to a volume of 250 mm^3. The theoretical results were compared with experimental data obtained for a stainless steel tube connected to a cavity exposed to the diaphragm of a Setra 237 pressure transducer. The pressure fluctuation amplitudes in the reference or coupling cavity, at the input end of the tube, were measured by a Bruel and Kjaer 4147 low frequency microphone. The amplitude of the sinusoidal pressure fluctuations in the coupling cavity was approximately 70 Pascals. However other runs with amplitudes varying from 20 to 200 Pascals showed no significant difference in measured response characteristics.

Reasonable agreement with the experimental data is obtained when the experimental tube diameter of 1.5 mm is used in the theoretical equations, including the resonant frequencies of the 2nd and 3rd modes. However a better agreement with the magnitude of the resonant amplitude peaks and phase response, is achieved when the tube diameter is reduced by 10% in the theoretical calculations. Bergh and Tijdeman [2] observed a similar phenomenon in their comparisons and attributed the discrepancy to inaccurate measurement of the tube diameter. We do not believe that this is the reason for the differences in our case, as great care was taken to accurately measure the internal diameter of the tube.

Other comparisons have been made for series tubes involving flexible (p.v.c.) tubes, changes in diameter due to pressure taps, and restrictors. Generally, the theory has predicted the experimental data quite well in these cases, although it is not always possible to specify the diameter of the p.v.c. tubing accurately.

Manifold Tube Comparisons

A comparison for a system involving a 10 input manifold system, as well as flexible tubing, pressure taps and restrictors is shown in Figure 2.This is a near optimum system, with a flat amplitude response (± 5%) and near-linear phase lag up to nearly 300 Hz, that has been used for measurement of area-averaged loads on wind tunnel models of buildings. For such a complex system, the theory predicts the measured response quite well. No reduction of tube diameters was made in this case.

It should be noted that the input tubes to the manifold were assumed to be of constant diameter, in the theoretical calculations. In fact, in the experiment there were pressure taps of smaller diameter at the start and termination of these tubes. Although it is possible to account for these in the theoretical model, the computations become considerably more complicated [6] and were not justified for the small improvement in accuracy to be achieved.

Discussion and Conclusions

Like Gumley [6], we believe that the theoretical method can be used to optimize tubing systems required for measurements. However, it is essential that an experimental calibration be carried out subsequently. When this is done, fine tuning of the system, by, for example, moving the restrictor(s) along the tube is usually required for final optimization.

It should also be noted that tubing systems incorporating a 'Scanivalve' pressure scanning switch cannot be adequately be included in the theoretical predictions. The interior passages of such devices include right angled bends, and tapered tubes which are not catered for by the theory. We have found that experimental trial and error calibration must be used for systems involving these devices.

A paper describing an optimization procedure for pressure systems is in preparation. However, for most cases, both single tube and manifolded systems, it is found that the best results are obtained using restrictors whose optimum positions are fairly close to the pressure transducer. Figure 2 shows the response curves for such a system.

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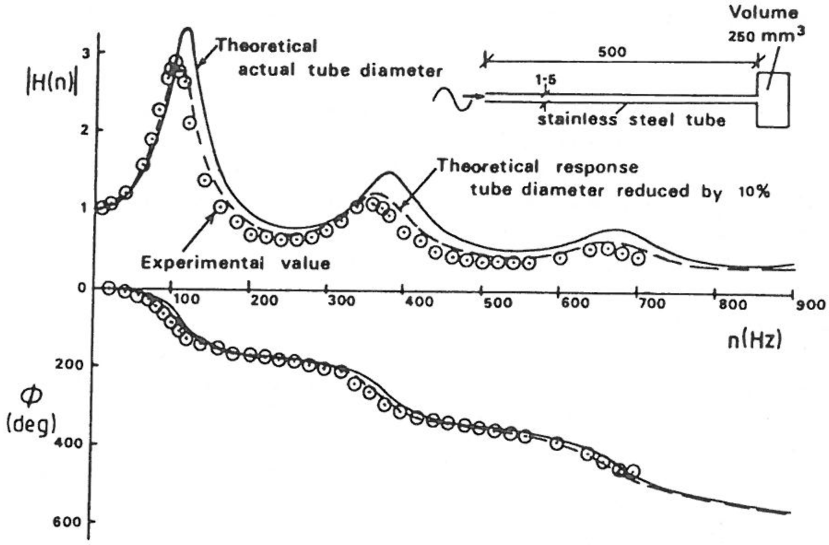

Figure 1. Comparison of responses - simple system

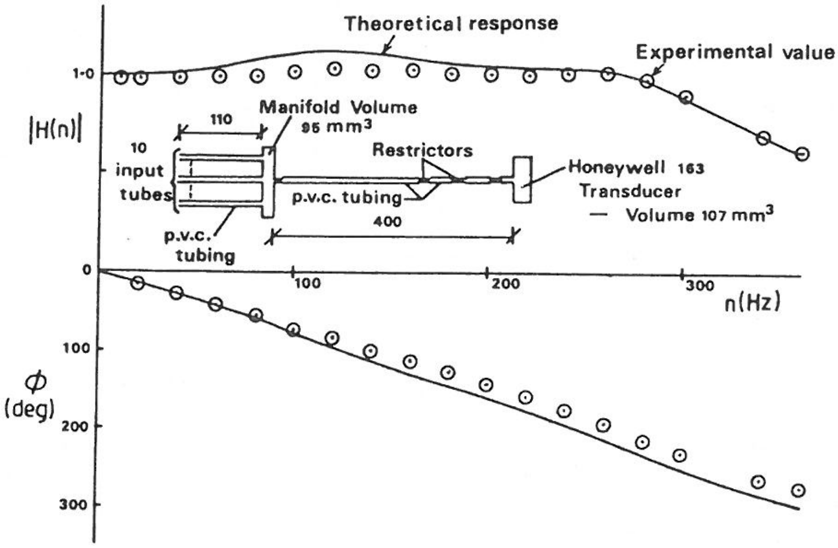

Figure 2. Comparison of responses - complex system