B.R. Morton \*

#### Introduction

The interaction of shear layers in flow past either surface-mounted or freestanding obstacles gives rise to complex three-dimensional secondary flows, which may be observed both upstream of the obstacles and in their wakes. flows have been quite widely studied, although the work is predominantly empirical and qualitative, but the flow behaviour remains poorly understood and difficult to predict. One obvious reason for this complexity is that the behaviour of flow past a cross-stream-symmetrical surface-mounted obstacle may depend on the gross height and width of the obstacle, representative radii of curvature in the vertical and horizontal at its leading edge, and the thickness of the incident boundary layer. Thus the flow may be characterised by as many as five Reynolds numbers, and we have only limited understanding of their relative importance. A surface mounted hemisphere might be regarded as one of the simpler types of obstacle as its representative lengths are all determined by the radius. Such hemispheres generate one or more pairs of trailing vortices in their wakes over a wide range of Reynolds numbers, whether the wakes be laminar or turbulent. In recent years there has been some disagreement as to whether the principal vortex pair has sense of rotation such as to produce upwash behind a hemisphere (or similar obstacle) or downwash. Although little of this discussion has been published, one opinion held that the sense depended on whether the flow was laminar or turbulent, Hanson [1] found predominant downwash in a wind tunnel study, while the results of Mason and Morton [2] suggest that downwash will dominate the wake when the ratio of boundary layer thickness to radius is small and boundary layer fluid is diverted primarily to the sides of the hemisphere, but that the wake vortices will be associated with upwash when this ratio is larger and diversion is primarily over the crest.

Much of the work on shear layer interaction with obstacles has related to the effects of small protuberances on boundary layer flows and on larger surface—mounted obstacles projecting through their boundary layers. The impetus for the former work arose primarily in aerodynamics and has been reviewed by Sedney [3], who noted that there had been little theoretical work and that insight into the complex flow patterns observed had come mainly from the visualisation of laboratory flows. One of the more detailed early studies was by Gregory and Walker [4], who described horseshoe shaped vortices which were seen wrapped around a low cylindrical obstacle, with arms trailing downstream. Studies of flow past taller obstacles have developed both in aerodynamics, where struts, fins and wings are believed to trap advected boundary layer vorticity, forming

<sup>\*</sup> Department of Mathematics, Monash University, Clayton, Vic. 3168, Australia.

horseshoe vortices round the bases of the obstacles (Baker [5]; Bradshaw [6]) and in civil engineering where it has long been accepted that strong horseshoe vortices due to the trapping of boundary layer vorticity by bridge piers are responsible for bridge pier erosion of bed material upstream of the piers (Qadar [7]).

#### The horseshoe vortex paradox

On the traditional view, illustrated in Figure 1(i), a horseshoe vortex forms round the base of a cylinder in the boundary layer where cross-stream filaments of boundary layer vorticity are trapped ahead of the cylinder but carried past on either side. These filaments cannot be cut, and therefore collect into a U-shaped vortex round the cylinder with arms trailing off downstream. Advective stretching of the arms which tends to reduce the vortex core diameter and diffusive thickening which tends to increase it then settle into a balance, producing a strong, steady-in-the-mean horsehoe vortex lying parallel to the lower boundary and roughly within the boundary layer.

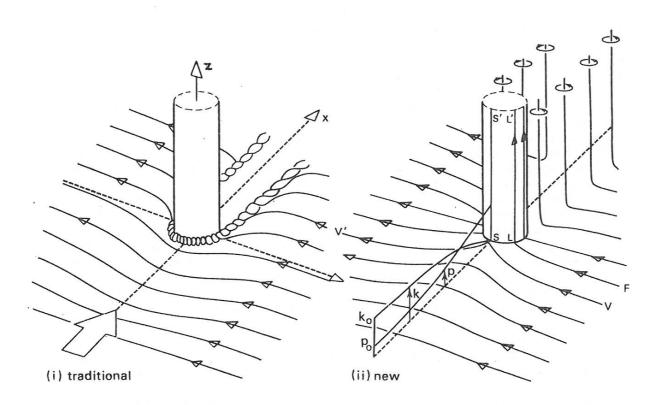


Figure 1 Schematic diagrams representing (i) the traditional and (ii) the proposed structure of vortex filaments where a boundary layer interacts with a cylinder. The diagrams represent a high Reynolds number, thin boundary layer limit.

The traditional view is unacceptable as it leads directly to a paradox. Suppose, in the configuration of Figure 1, a stream with uniform undisturbed velocity U in the x-direction and a boundary layer over the lower boundary interacting with a vertical cylinder mounted on the boundary. A measure of the gross vorticity in the boundary layer is provided by the circulation per unit streamwise length through the full depth of the layer, U; and a measure for the

mean speed of vorticity advection in the boundary layer is close to  $\frac{1}{2}U$  for all likely profiles of velocity in the layer. It follows that the flux of vorticity in the boundary layer towards the cylinder has measure  $\frac{1}{2}U^2$ , and that the trapped circulation after time t is approximately  $\frac{1}{2}U^2t$  and is unbounded! Moreover, this paradox cannot be resolved by viscous diffusion since in the upstream plane of symmetry all vorticity has the sense of that being advected in the boundary layer; nor can the trapped vortex filaments be advected up over the top of the cylinder, as no corresponding disturbance is observed on the upstream face of cylinders extending far above the boundary layer.

# The interaction of a boundary layer with a cylinder

Further insight may be gained by flow visualisation of a stream U flowing over a horizontal lower boundary on which is mounted a vertical circular cylinder which extends through the boundary layer into the outer stream. Figure 2 shows a time exposure in the upstream plane of symmetry for flow past a circular cylinder. The flow is in a water channel with  $0.2\times0.2\text{m}^2$  section in which the water is seeded with very small mica platelets and illuminated with a thin vertical sheet of light. Suitably oriented platelets reflect light horizontally to the camera, giving lines in the time exposure, but the platelets rotate in shear and so give streamlines and an indication only of velocity. The cylinder, just visible to the right, has height/diameter ratio h/d=0.36, height/boundary-layer-thickness ratio about 2, and Reynolds number 2700 (=Ud/v). The outer edge

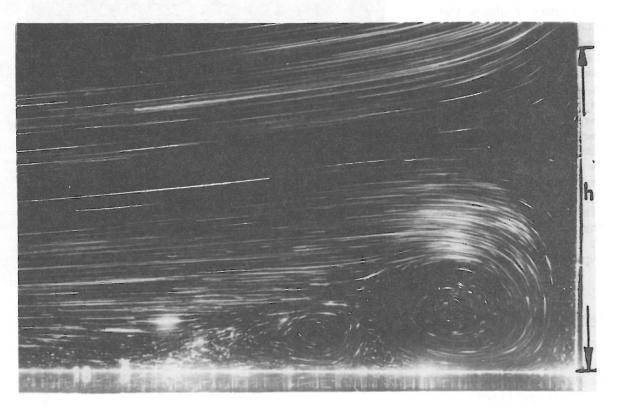


Figure 2 A visualisation in the upstream plane of symmetry for flow past a circular cylinder with h/d=0.36,  $h/\delta\approx 2$  and Re=2700.

of the boundary layer in Figure 2 corresponds to the stagnation point at about 3h/4, where h is the height of the cylinder. Above this stagnation point the flow is deflected over the top of the cylinder, while below it is a complex pattern of flow comprising in sequence from the right a large vortex with positive

sense of rotation (that of the boundary layer), a small lower "triangular" vortex with negative sense of rotation, an intermediate vortex of positive sense, and a flow separation which is poorly marked on account of the small speeds near the stagnation point.

The first point that should be made is that, although U-shaped vortices are certainly present round the base of the cylinder, they are surprisingly weak, with flow speeds far below those of the outer boundary layer and stream and vorticity well below that of the boundary layer.

Secondly, the small triangular vortex between the two larger positive (clockwise) vortices is quite definitely anticlockwise or negative in sense, and therefore consists of vorticity present nowhere else in this symmetry plane. For symmetry reasons, this vorticity cannot have been produced by turning boundary layer vorticity through angle  $\pi$ . The pattern is weaker and more complex than the anticipated classical horseshoe vortex and contains negative vorticity: it is therefore incompatible with the classical horseshoe model.

This and other flows will be discussed at the meeting, especially in terms of the generation of vorticity at the boundaries and the proposed model of Fig.l(ii).

# Wake and cylinder

Figure 3 shows the interaction of a wake generated from a splitter plate 40mm wide and 80mm upstream of the cylinder at Reynolds number 1200. In this case the advected vorticity is positive above and negative below the plate level. There is stretching as the flow is diverted past the cylinder resulting in a visible vortex pair, but insufficient generation at the cylinder for flow separation there.

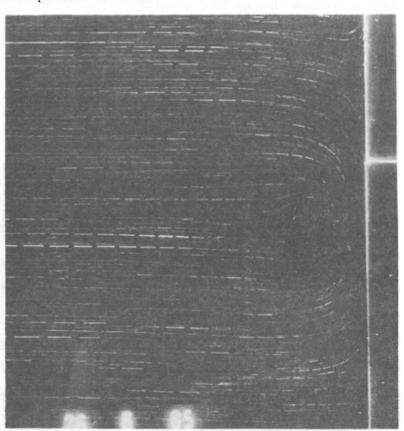


Figure 3 Interaction of wake and cylinder.

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