AEROELASTIC ANALYSIS OF SAILS

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Introduction

The pressure distribution acting on a lifting surface depends on its shape, and usually this shape is fixed - that is, the aerodynamic loads do not cause significant changes in the shape of the lifting surface. However the shape of a sail under load depends strongly on the distribution of pressure over it, so that the shape determines the pressure but the pressure is determined by the shape. Therefore in order to predict this shape from first principles it is necessary to solve the aerodynamic and structural problems simultaneously. One way of doing this is to split the problem into two - that is, first assume a fixed shape for the sail and calculate the pressure distribution acting on it, then hold the pressure fixed and calculate the shape taken up by an elastic membrane under the action of the pressure loads, then repeat the whole process. This is the basis of a successful solution method which has been developed by the author (Jackson, 1982, 1985) together with his colleague Dr G.W. Christie.

Prediction of Sail Shape

The sail is modelled as a surface of constant-strain triangles connected at the nodes, allowing the state of strain of each triangle to be expressed in terms of the displacements of its nodes. If the nodes are displaced from an equilibrium position by a small distance, the virtual work done by the forces at the nodes is exactly balanced by the change in strain energy of the elements, and this statement can be expressed as a nonlinear relationship between the sail shape, the applied pressures, the material properties and the unknown nodal displacements. A detailed derivation of these equations is given by Oden and Sato (1969).

The calculation of the strain energy requires some statement about the stress/strain relationship of the sail fabric. As the strains in the sail membrane are small (not exceeding 0.5%), a linear constitutive law can be used. It is convenient to suppose that the fabric is isotropic. While it is not difficult to account for this, here only isotropic behaviour is assumed so that the elastic modulus E is all that is needed to describe the cloth stiffness. It is also necessary to model the inability of real membranes to withstand compressive in-plane stress - a membrane simply buckles, or wrinkles.

Dimensional analysis suggests that an important ratio is the aeroelastic number

$$\mathcal{E} = E\, t/L\, q \ ,$$

where q is the dynamic wind pressure, and t/L the ratio thickness to chord of the sail. Jackson (1985) has shown that this number represents the ratio of the elastic stiffness of the sail to its aerodynamic stiffness (the rate of change of aerodynamic force with sail displacement). In practice this number is large (at least 10^3), so that there is no need to actually calculate this aerodynamic stiffness. The physical significance of the

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number is that any two sails with the same value of Æ and the same boundary and upstream conditions will experience the same strains and generate the same lift coefficients at the same angle of attack.

As the system of equations for the nodal displacements is highly nonlinear, it is solved iteratively using the Newton–Raphson method. The applied pressures vary as the solution proceeds, and progressively approach the correct values as the membrane solution approaches its final shape. The solution method is discussed in more detail by Jackson (1985).

Pressure Prediction

The pressure forces are calculated using potential flow theory, in which boundary layers are effectively regarded as vortex sheets having zero thickness but with a finite jump in velocity across them. The sheet strength is found by requiring the sum of the flow induced by the sheet and the incident flow to be tangential to the sail surface. Since the finite element model needs only the average pressure on each element it is convenient to use a boundary element method in which the vorticity contained within each element is concentrated around its edges as a vortex loop – the vortex sheet then becomes a so-called vortex lattice. The strength of the loops is then found by applying the zero normal flow condition at the center of each element, giving as many equations as unknowns. This technique is well established in conventional aerodynamics.

The solution obtained is not unique unless the points at which the flow separates from the sail surface to form the wake are defined. The only situation for which this can be done accurately is that where the flow leaves the trailing edge of the sail smoothly, and this 'Kutta' condition is easily applied by extending the vortex loops adjacent to the trailing edge so that they trail off to infinity downstream of the sail.

The presence of the water surface is accounted for by including the image of every vortex loop in the surface. The approaching freestream flow is actually sheared and twisted by the vectorial addition of the wind boundary layer and the boat velocity, so that some vertical displacement of streamlines occurs as the wind approaches the sail. This displacement is ignored, but could be accounted for by yet more iterations. As there are no viscous effects included it is not possible to predict drag arising from skin friction, or the onset of stall.

Results for a Simple Sail

A rather idealised sail has been chosen as an example. The sail is triangular with no gap under the foot. It has no pretensioning and no initial camber, so in the absence of wind loading it is perfectly flat. The elastic constant for the membrane is Æ = 1000, which means that if a 2-D membrane were loaded uniformly with a pressure of q it would deflect laterally to a displacement of about 4% of the chord. The corner nodes of the sail are totally constrained, as are those on the leading edge (luff). The trailing edge (leech) nodes are not restrained at all.

Figure 1 shows the equilibrium shape of the sail under wind loading at six equi-spaced heights for an angle of attack of 10°. Since the flat sail must stretch under the action of *any* transverse loading, the loaded sail has acquired both camber and twist. Aerodynamic loading is strongly concentrated near the leading edge (it reduces to zero at the leech) so the camber tends to be larger there. However because the leech is unconstrained it has little resistance to transverse loads and thus allows the sail sections to twist so as to reduce the angle of attack. Initially this effect is very pronounced but once the sail has acquired some geometric stiffness by virtue of curvature along the leech between the two restrained corners it becomes progressively more resistant to lateral loading. This is reflected in the change in lift coefficient with angle of attack, as shown in Figure 2.

The twist developed by the membrane sail means that its lift is less than that of a rigid sail of the same initial shape, which is also shown. The stresses in this sail for an angle of attack of 10 degrees are shown in Figure 3. Here the line segments are proportional to the principal stresses of the elements, and lie in the same directions. It can be seen that in many cases the elements do not show a minor principal stress, which means that they are wrinkled. If the aeroelastic number is now reduced by a factor of five the sail develops much more twist, as shown by the rear views of the sails for the two levels of stiffness shown in Figures 3 and 4. If this reduction in Æ has occurred because the sail is more stretchy then the levels of stress are also reduced because less lift and more curvature are developed - this can also be seen in these figures. However if the sail is the same but Æ is reduced by an increase in dynamic pressure, then the stresses as plotted must be increased by the same factor - this leads to an increase in stress, as expected. To demonstrate the effect of boundary conditions the edge restraints are now altered so that both the luff and foot slide along their respective axes. The stress distribution produced is that shown in Figure 5. While this is markedly different from Figure 3 the levels of indued camber and twist are about the same, as expressed by the small difference between lift coefficients for the two sails (Fig 2). The final example is more realistic shape with initial vertical and horizontal camber. Because the unloaded shape has curvature it also has a greater geometric stiffness, but as the same camber also greatly increases the lift the net level of stress is about the same as before. Figure 6 shows that the stress now tends to be concentrated along the leech.

Conclusions

One of the obstacles to rational sail desgin has been the absence of any means of engineering analysis. While it will now be possible to estimate the structural and aerodynamic performance of any given sail, it is hoped that this method will identify the most important of the numerous variables in sail design and thus lead to the development of a simpler model which can be used to predict optimum sail shapes directly.

References

Jackson, P.S. (1982); *'A 3-D aeroelastic sail model'*, Science of Sail Design, conf. proc. University of Western Ontario Boundary Layer Wind Tunnel Laboratory, June.

Jackson, P.S. (1985); *'The analysis of three-dimensional sails'*, CANCAM proceedings,

Oden, J.T. and T. Sato (1967); *'Finite strains and displacements of elastic membranes by the finite element method'*, Int. J. Solids Structures, 3, 471-488.

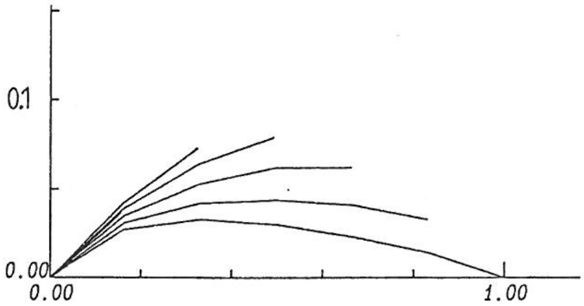

Figure 1 : Sections of loaded sail (Flat-1), initially flat.

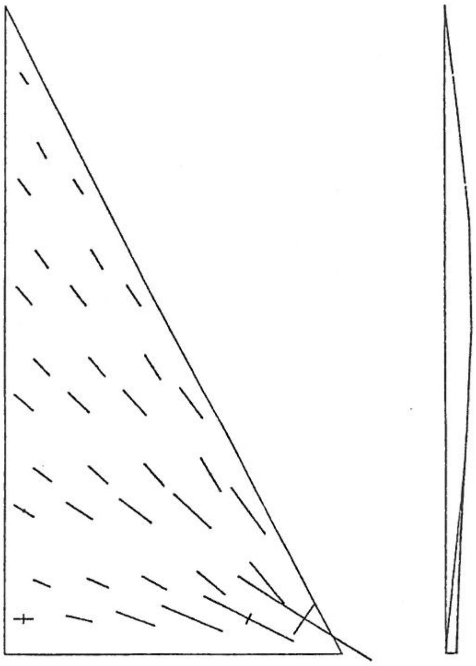

Figure 3 : Shape and stress of initially-flat sail (Flat-I) - fixed luff, Æ = 1000.

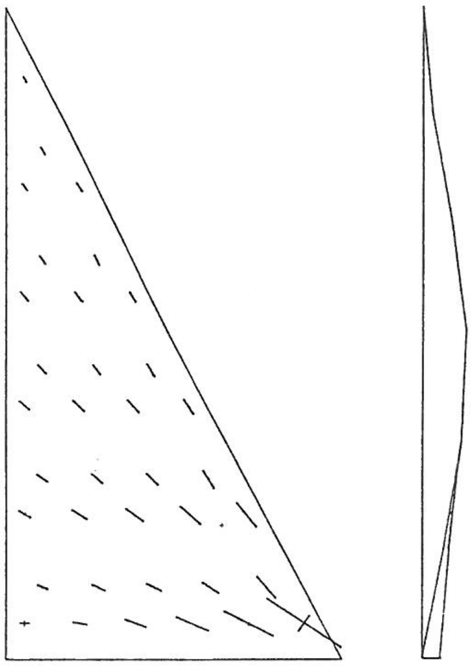

Figure 4 : Shape and stress of initially-flat sail (Æ = 200).

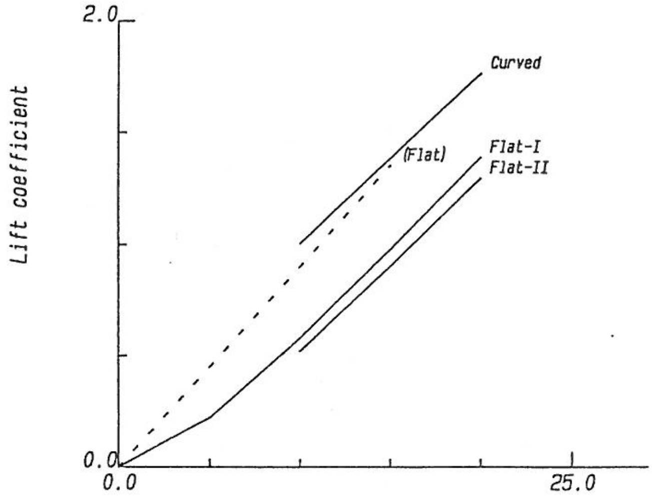

Figure 2

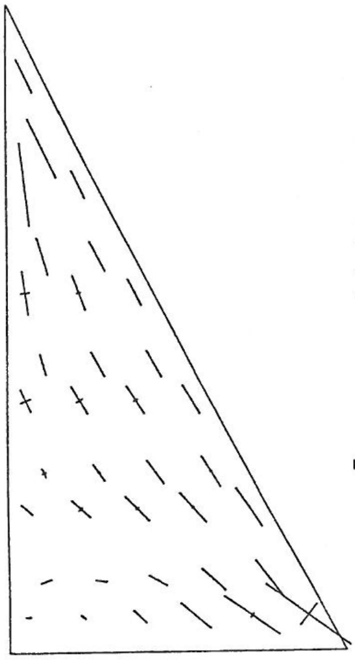

Figure 5 : Stress of initially-flat sail (sliding edges)

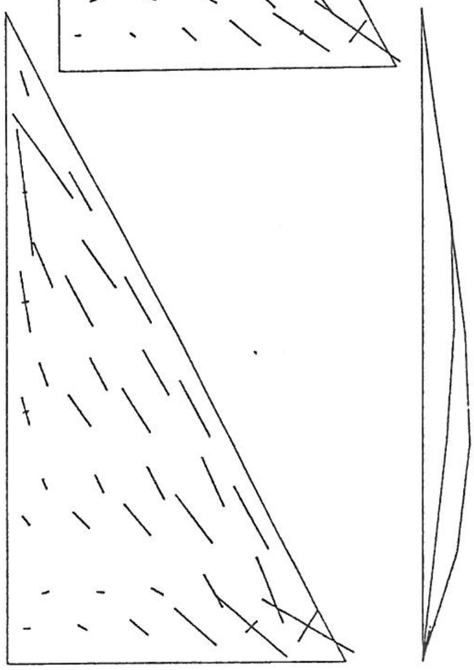

Figure 6 : Shape and stress of initially-curved sail.